Applications of Autonomous Computational Methods for Finding Step-by-Step Solutions

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Outline

1. Online Learning
2. Automatically Generated Examples
3. New Computational Methods
4. Self-Adaptive and Autonomous Computational Methods
5. Conclusions

Online Learning

TA Homework

	Return to my homepage
Username:	ta
First Name:	Andrew
Last Name:	Pownuk
Group:	2018-Spring-MATH-4329-CRN-24656
	Logoff

Number of homework: 12

Create new Homework

homework-description	Homework 13
homework-id	Homework-13
link	homework/Homework-13.aspx
tries-max	10

Online Learning

- Sharing the lecture notes.
- Interactive platform for doing online homework.
- Automated system for checking attendance.
- Integrated response system.
- Grades management system.
- Interactive projects.

Automatically Generated Examples

- It is necessary to explain as basic mathematical/scientific concepts in details.
- Student's can compare their work with available examples.
- Automatically generated examples can be used as sample assignments in on-line homework, exams, and ungraded assignments.
- Automatically generated examples can be related to many different methods for solutions.
- **It is possible to crated millions examples in a very short time**.
- **If all inference rules are done properly, then the examples are without errors.**.

Example (differentiation)

- Table of basic formulas for differentiation.
- Product rule, quotient rule, chain, rule etc..
- Rules for simpling expression.
- Automatically generated examples can be related to many different methods for solutions.
- Knowledge available to the system is in the format which is almost exactly **the same like in the regular textbooks** (no machine learning ...).

Example (differentiation)

Sample formulas

$$\cos(x) + \cos(x) + 2 + \sin(\sin(x)) \tag{4591}$$

$$\cos(x) + \cos(x) + 2 + \sin(\cos(x)) \tag{4592}$$

$$\cos(x) + \cos(x) + 2 + \cos(2) \tag{4593}$$

$$\cos(x) + \cos(x) + 2 + \cos(x) \tag{4594}$$

$$\cos(x) + \cos(x) + 2 + \cos(\sin(x)) \tag{4595}$$

$$\cos(x) + \cos(x) + 2 + \cos(\cos(x)) \tag{4596}$$

Example

Sample results

$$\cos(x) + -1 \cdot \sin(x) + -1 \cdot \sin(\sin(\sin(x))) \cdot \cos(\sin(x)) \cdot \cos(x) \tag{3794}$$

$$\cos(x) + -1 \cdot \sin(x) + -1 \cdot \sin(\sin(\cos(x))) \cdot \cos(\cos(x)) \cdot -1 \cdot \sin(x) \tag{3795}$$

$$\cos(x) + -1 \cdot \sin(x) + -1 \cdot \sin(\cos(2)) \cdot -1 \cdot \sin(2) \cdot 0 \tag{3796}$$

$$\cos(x) + -1 \cdot \sin(x) + -1 \cdot \sin(\cos(x)) \cdot -1 \cdot \sin(x) \tag{3797}$$

$$\cos(x) + -1 \cdot \sin(x) + -1 \cdot \sin(\cos(\sin(x))) \cdot -1 \cdot \sin(\sin(x)) \cdot \cos(x) \tag{3798}$$

$$\cos(x) + -1 \cdot \sin(x) + -1 \cdot \sin(\cos(\cos(x))) \cdot -1 \cdot \sin(\cos(x)) \cdot -1 \cdot \sin(x) \tag{3799}$$

Example (Latex source)

```
\begin{equation}\cos\left(x\right)+x+2+\sin\left(2\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(\sin\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(\cos\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(2\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(\sin\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(\cos\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+2\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+x\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+x+x\end{equation}
\begin{equation}\cos\left(x\right)+x+2+x+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+x+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(x\right)+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(x\right)+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(x\right)+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+x+x\end{equation}
```

Step by Step Solution

Wolpharm Alpha

Indefinite integrals:

$$\int x \exp(x)\, dx = e^x (x-1) + \text{constant}$$

Possible intermediate steps:

Take the integral:

$$\int e^x x\, dx$$

For the integrand $e^x x$, integrate by parts, $\int f\, dg = f g - \int g\, df$, where

$f = x, \quad dg = e^x\, dx,$
$df = dx, \quad g = e^x$:

$$= e^x x - \int e^x\, dx$$

The integral of e^x is e^x:

$$= e^x x - e^x + \text{constant}$$

Differentiation (step 1)

$$\cos(x) + (\cos(x) + \cos(2)) \tag{765}$$

$$\cos(x) + (\cos(x) + \cos(x)) \tag{766}$$

$$\cos(x) + (\cos(x) + \cos(\sin(x))) \tag{767}$$

$$\cos(x) + (\cos(x) + \cos(\cos(x))) \tag{768}$$

$$\cos(x) + (\cos(x) + (2 + 2)) \tag{769}$$

$$\cos(x) + (\cos(x) + (2 + x)) \tag{770}$$

$$\cos(x) + (\cos(x) + (2 + \sin(x))) \tag{771}$$

$$\cos(x) + (\cos(x) + (2 + \cos(x))) \tag{772}$$

$$\cos(x) + (\cos(x) + (x + x)) \tag{773}$$

Differentiation (step 2)

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}(\cos(x) + \cos(2)) \tag{765}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}(\cos(x) + \cos(x)) \tag{766}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}(\cos(x) + \cos(\sin(x))) \tag{767}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}(\cos(x) + \cos(\cos(x))) \tag{768}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}(\cos(x) + (2 + 2)) \tag{769}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}(\cos(x) + (2 + x)) \tag{770}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}(\cos(x) + (2 + \sin(x))) \tag{771}$$

Differentiation (step 3)

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\sin(x)\right) \tag{762}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\sin(\sin(x))\right) \tag{763}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\sin(\cos(x))\right) \tag{764}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\cos(2)\right) \tag{765}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\cos(x)\right) \tag{766}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\cos(\sin(x))\right) \tag{767}$$

Online Learning

Automatically Generated Examples

New Computational Methods

Self-Adaptive and Autonomous Computational Methods

Conclusions

Differentiation (step 4)

$$(-1)\cdot\sin(x)+\left((-1)\cdot\sin(x)+\left(\frac{d}{du}\cos(u)\right)_{u=\sin(x)}\cdot\frac{d}{dx}\sin(x)\right) \tag{767}$$

$$(-1)\cdot\sin(x)+\left((-1)\cdot\sin(x)+\left(\frac{d}{du}\cos(u)\right)_{u=\cos(x)}\cdot\frac{d}{dx}\cos(x)\right) \tag{768}$$

$$(-1)\cdot\sin(x)+\left((-1)\cdot\sin(x)+\left(\frac{d}{dx}2+\frac{d}{dx}2\right)\right) \tag{769}$$

$$(-1)\cdot\sin(x)+\left((-1)\cdot\sin(x)+\left(\frac{d}{dx}2+\frac{d}{dx}x\right)\right) \tag{770}$$

$$(-1)\cdot\sin(x)+\left((-1)\cdot\sin(x)+\left(\frac{d}{dx}2+\frac{d}{dx}\sin(x)\right)\right) \tag{771}$$

$$(-1)\cdot\sin(x)+\left((-1)\cdot\sin(x)+\left(\frac{d}{dx}2+\frac{d}{dx}\cos(x)\right)\right) \tag{772}$$

$$(-1)\cdot\sin(x)+\left((-1)\cdot\sin(x)+\left(\frac{d}{dx}x+\frac{d}{dx}x\right)\right) \tag{773}$$

Differentiation (step 5)

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+\cos(\sin(x))\cdot\cos(x)) \tag{763}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+\cos(\cos(x))\cdot(-1)\cdot\sin(x)) \tag{764}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+0\cdot 0) \tag{765}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(-1)\cdot\sin(x)) \tag{766}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(-1)\cdot\sin(\sin(x))\cdot\cos(x)) \tag{767}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(-1)\cdot\sin(\cos(x))\cdot(-1)\cdot\sin(x)) \tag{768}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+0)) \tag{769}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+1)) \tag{770}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+\cos(x))) \tag{771}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+(-1)\cdot\sin(x))) \tag{772}$$

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(1+1)) \tag{773}$$

New Computational Methods

Available information.

- Linear equation with one variable.

$$2 + x = 4$$

- Solution

$$x = 2$$

- Knowledge base of mathematical operations, theorems, etc.

Solution

Solution procedure computed automatically based on available knowledge.

- $2 + x = 4$
- $x + 2 - 2 = 4 - 2$
- $x + 0 = 2$
- $x = 2$

Now it is possible to create new solution procedure based on presented algorithm and use it for solution of other problems, etc.

Integration

Problem

- $\int (2x + 1) dx$
- Rules for integration, theorems, algebraic rules, etc.

Solution

- $\int (2x + 1) dx$
- $\int 2x dx + \int 1 dx$
- $2 \int x dx + x$
- $2 \frac{x^2}{2} + x + C$

Now it is possible to implement this method as a new procedure and use it in the future.

Self-Adaptivity

- The system is fully autonomous. All changes in the program can be done automatically without interaction with the user.
- All changes of the code of the program can be done during the runtime.
- System is distributed and can work independently on many computers which improve reliability of the system.
- Once information is added to the system it will never be forgotten and can be reused in the future in order to create new (improved) knowledge.
- Presented methodology can be applied not only to mathematical problems but also to any other scientific field which can be described by some abstract concepts (e.g. statistics, engineering, chemistry, biology, computer science etc.).

Imaginary Universe of Scientific Knowledge

"Imaginary Universe" terms used by some MIT researchers.
Quantum Artificial Life in an IBM Quantum Computer
(U. Alvarez-Rodriguez, M. Sanz, L. Lamata & E. Solano).
Scientific Reports volume 8, Article number: 14793 (2018)

- Mathematical/scientific knowledge can be treated as independent units that can interact with each-other and create new, possibly useful knowledge.
- Generation of new knowledge can be fully automated and autonomous. No interaction with humans is necessary.
- Development of new knowledge is possible in many different fields (e.g. statistics, engineering, chemistry, biology, computer science etc.).

Conclusions

- By using presented methodology it is possible to create complex educational examples in many areas of mathematics as well as in other areas of science and engineering.
- In a few minutes it is possible to create thousands pages with typical examples without interaction with humans and that can be used in education.
- By using self adaptive computational methods it is possible to automatically generate new mathematical theorems completely independently from human interactions.