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Dynamic Analysis of a Cable-Stayed Bridge with Uncertain Structural Parameters

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ABSTRACT: This paper focuses on the transient dynamic analysis of a cable-stayed bridge with uncertain structural parameters, subjected to an impact load. The analysed uncertainty is associated with the Young's modulus and mass density of the reinforced concrete deck slab. These parametric uncertainties are quantified based on the fuzzy formalism, and the uncertain transient analysis is performed using the alpha-sublevel technique. At the core of the analysis, Wilson's theta method is applied to solve the transient response problem. In order to solve the sequence of optimisation problems, a Kriging-based response surface methodology adapted to multiple output analysis of FE models is developed. The performance and accuracy of this approach is examined with reference to a classical global and local direct optimisation approach. The focus in this comparison is both on the accuracy in the obtained interval results and on the computational efficiency. The work demonstrates the effectiveness of the Kriging-approach based fuzzy finite element method in evaluating the dynamic response of the cable-stayed bridge with multiple uncertainties.

1 INTRODUCTION

Analysis and design of structures occupy an important place in the field of structural mechanics. Modern day structures are usually complex in geometry and are made of a combination of several materials. In order to ensure that structures do not fail during their intended design life period with catastrophic and unpredictable consequences, proper analysis and design are mandatory.

The finite element method has become a very popular tool for design validation and optimisation. Continuously growing computational capabilities allow for extremely detailed numerical models of structural systems. However, the computational power could be of much greater value when used for the inclusion of uncertainties in the model rather than for modelling deterministic details. This brings out the need for the development of non-deterministic finite element modelling techniques.

The non-deterministic approaches are gaining momentum in the field of numerical modelling techniques. The ability to include non-deterministic properties is of great value for a design engineer. It enables realistic reliability assessment that incorporates the uncertain aspects of the design. Furthermore, the design can be optimized for robust behaviour under varying external influences.

Recently, criticism has arisen regarding the general application of the probabilistic concept in this context. Especially when objective information on the uncertainties is limited, the subjective probabilistic analysis result proves to be of little value, and does not justify its high computational cost (Elishakof 2000). Consequently, alternative non-probabilistic concepts have been introduced for non-deterministic numerical modelling (Moens & Vandepitte 2005). In this context, alternative concepts like the interval and fuzzy approaches are becoming increasingly popular for the analysis of numerical models that incorporate uncertainty in their description.

One approach to handle uncertainty in a structural system is the use of interval algebra. Though interval arithmetic was introduced by Moore already in 1966, the application of interval concepts to structural analysis is more recent. Rao & Sawyer (1995) and Rao & Chen Li (1998) have developed different versions of interval-based finite element methods to account for uncertainties in engineering problems.

The fuzzy approach extends the interval methodology by introducing a level of membership that represents to what extent a certain value is member of the range of possible input values. This concept provides the analyst with a tool to express a degree of possibility for a certain value. Based on the α-sublevel technique, the fuzzy analysis requires the consecutive solution of a number of related interval problems. Owing to this, much attention goes to the actual solution and implementation of interval analysis.

Extensive research helped understanding the behaviour of imprecisely defined systems using fuzzy arithmetic. Mullen & Muhanna (1999) developed a fuzzy-based matrix method of structural analysis for the calculation of extreme values of structural response for all possible loading combinations. More recently, Muhanna & Mullen (2001) handled uncertainty in mechanics problems by using an element-by-element technique to obtain a sharp enclosure for the fuzzy solution by eliminating the sources of overestimation. Muhanna et al (2005) dealt with a penalty-based solution for interval finite element methods to obtain a sharp enclosure of fuzzy solutions in comparison with the exact solution.

A practical approach for analyzing the structures with fuzzy parameters was also developed by Akpan et al (2001a). The uncertainties in material, loading and structural properties were represented by convex normal fuzzy sets. A vertex solution methodology that was based on an α-cut representation was used for the fuzzy analysis. Response surface methodology and combinatorial optimisation were used to determine the binary combinations of the fuzzy variables that resulted in fuzzy responses at an α-cut level. These binary combinations of the fuzzy variables were then used to obtain extreme responses to the finite element model.

However, not much work was carried out in determining the effect of uncertainty in structural parameters on the transient dynamic response of a structural system. Most of the literature dealing with the application of fuzzy finite element methods for structural dynamic analysis focuses on frequency-domain based techniques (Moens et al. 2004). The objective of this work is to develop a methodology for obtaining the transient response of a cable-stayed bridge subjected to a sudden impact load, considering uncertainty in the Young's modulus and mass density of concrete and steel. The Centre Canal Bridge at Obourg, Belgium was chosen as an example problem. The cable-stayed bridge was used earlier in the context of fuzzy finite element analysis, i.e. for a study of the effect of live load on the static displacements of the bridge (Rama Rao and Ramesh Reddy 2007).

After a general introduction, section 2 discusses implementation techniques for the interval finite element method. Section 3 then focuses on the application of the Kriging-based response surface methodology for numerical problems with multiple outputs. The numerical case study is presented and analyzed in section 4. Finally, section 5 gives general conclusions regarding the application of the presented methodology for transient time domain structural dynamic analysis.

2 IMPLENTATION OF INTERVAL FINITE ELEMENT ANALYSIS

2.1 *Interval finite element analysis*

In recent literature, the application of both the interval and the fuzzy concept for the representation of parametric uncertainty during a classical finite element analysis has been studied extensively. While the problem at the core of the analysis, i.e. the solution of a set of interval equations, is easily formulated, the actual solution of this problem was proven to be extremely problematic (Moens & Vandepitte 2005). Nevertheless, some solution schemes of fundamentally different nature have been developed.

Interval finite element analysis is based on the interval concept for the description of non-deterministic model properties. The goal of the interval finite element analysis is to obtain the range of specific output quantities that corresponds to a given interval description of the uncertainty on some input parameters of the problem. Numerically, this means that the solution procedure should focus on finding the minimal and maximal deterministic analysis results taking into account all possible models that are located within the interval uncertainty description. Consider the vector x of uncertain parameters defined to be contained within an interval vector (or hypercube) x^I . If the input-output relationship between these parameters and the output quantity of interest is represented by the function $f(x)$ applied on these parameters, the interval finite element procedure in case of transient dynamic analysis is numerically equivalent to finding the following solution set:

$$y^S(t) = \left\{ y(t) \middle| (x(t) \in x^I(t))(y(t) = f(x(t))) \right\} \quad (1)$$

The outcome of the interval analysis here is expressed as a set, as in general, it cannot be described exactly by an interval or hypercube. Furthermore, the transient analysis causes this set to be time-dependent. The correct interpretation of this expression is that the set y^S contains all response vectors y that are obtained at a certain time from applying the function $f(:)$ on all possible vectors within the interval vector x^I .

An exact description of this solution set is in many cases extremely difficult to find. Generally, however, only the individual ranges of some components of the result vector y^S are really of interest. Therefore, most research focuses on calculating an interval vector that approximates the exact solution set, but neglects the interdependencies between the output vector components. This is referred to as a hyper cubic approximation of the result. It describes a range for each vector component, but not all combinations of vector components within these ranges are part of the exact solution set. Ideally, the approximation should be the smallest hypercube around the exact solution set. If the result is used for reliability analysis, the hypercube should be conservative.

Apart from the diversity caused by the nature of the numerical problem at hand, a clear distinction can be made between two fundamental approaches for tackling the interval finite element solution. The interval arithmetic strategy approaches the exact hyper cubic circumscription of the interval result from outside. It is based on the calculation of guaranteed outer bounds. The global optimisation approach on the other hand calculates an inner approximation, as it searches the extreme output values of the solution set by performing an anti-optimisation procedure within the limits posed by the uncertain input interval space. While the interval arithmetic approach proves to be computationally less expensive than the optimisation approach, it very often results in a huge overestimation of the actual interval result, due to the dependency problem. The optimisation approach forms the basis of the Kriging-based response surface methodology, introduced in this paper.

2.2 *Optimisation based interval finite element analysis*

The optimisation approach determines the lower and upper bound on the output of a classical finite element analysis by performing a search algorithm inside the domain defined by the interval inputs. If this search is successful (i.e., the global minimum and maximum of the analysis result are found), it returns the smallest hypercube around the solution set expressed in Equation 1. As such, the solution of an interval finite element problem becomes an optimisation problem, the goal function of which is the deterministic finite element analysis. The uncertain parameters are the design variables, constrained by the interval vector delimiting their ranges. The optimisation is performed independently on every element of the result vector y .In case of transient dynamic analysis, the response vector y at a certain time t^* can be obtained as

$$\left\{y(t^*)\right\} = \begin{Bmatrix} y_1^I(t^*) \\ y_2^I(t^*) \\ \\ \\ y_n^I(t^*) \end{Bmatrix}, \quad \begin{aligned} \underline{y}_i(t^*) &= \min_{x \in x^I} f_i(x(t^*)),\ \text{i=1,...n} \\ \overline{y}_i(t^*) &= \max_{x \in x^I} f_i(x(t^*)),\ \text{i=1,...n} \end{aligned} \qquad (2)$$

Contrary to the interval arithmetic approach, the optimisation strategy approaches the interval result from the inside. This means that it does not guarantee conservatism, unless the actual bounds on the goal function are found. Furthermore, as the behaviour of the goal function with respect to the uncertain parameters is rather unpredictable, the corresponding computational effort for finding the optima in general is strongly problem dependent.

However, both these disadvantages are countered by the fact that this approach intrinsically keeps the uncertain parameters coupled throughout the analysis. Consequently, the optimisation strategy is immune to the excessive conservatism observed in the interval arithmetic approach. It is also observed that the goal function very often exhibits a smooth behaviour with respect to the uncertain parameters, which strongly facilitates the search for the global extreme values over the design space. Therefore, the optimisation strategy is more and more acknowledged as the standard procedure for performing interval finite element analysis. Research in this direction has mainly focused on efficient procedures for performing the search expressed in Equation 2. In this domain, different approaches can be distinguished:

Global optimisation: The global optimisation procedures perform the search for the exact bounds on the goal function by iteratively evaluating the goal function at designated points in the search domain. As one of the pioneers of fuzzy finite element modelling, Rao et al. apply a directional search based algorithm to tackle the optimisation (Rao & Sawyer 1995 and Rao & Chen 1998). Other global optimisation techniques often encountered in the framework of interval finite element analysis are linear programming (Köylüöğlu & Elishakoff 1998) and genetic algorithms (Möller et al 2000). Recently, the innovative $G\alpha D$ (Gradual α Decreasing) algorithm (Degrauwe 2007) was introduced in the context of fuzzy finite element analysis.

Response surface methods: In this approach, the goal function of the optimisation problem is approximated by an appropriate surrogate response function. The optimisation then is performed on this response function. The response surface methodology was first applied in the context of interval finite element analysis by Akpan et al. (2001 b). The main advantage of the approach consists of the avoidance of an exact goal function evaluation at each iteration point of the search algorithm, which can be very costly. On the other hand, it is clear that the accuracy of the approach relies completely on the exactness of the approximation of the response function. Section 3 of this paper will elaborate further on the possibilities of the response surface approach for fuzzy numerical analysis with multiple outputs.

3 KRIGING ALGORITHM

Application of the global optimisation strategy to solve the interval equivalent of a transient time-domain FE analysis requires the solution of a very high amount of optimisation problems: two for each considered time step. Furthermore, for fuzzy analysis, this optimisation has to be repeated on all α-levels of interest. Each upper or lower bound of the FE analysis result at a certain time is calculated using a global optimisation run on the finite element model. The output value of the transient analysis at the analysed time-step constitutes the goal function of the optimisation problems. The inputs to this function are the uncertain parameters, which are bounded in the problem description. Because of the high number of optimisation runs required - e.g. a transient fuzzy analysis performed on 5 α-levels, taking into account 100 time-steps requires already 1000 optimisation runs – an efficient optimisation algorithm is absolutely necessary.

Generic non-linear optimizers can solve all optimisation problems independent of each other. Theoretically, the optimisation problems can be non-convex, requiring global optimisation software. In practice however, physical output quantities (as e.g. displacements and stresses) tend to have a rather smooth behaviour with respect to the uncertain physical input quantities. In many cases, many of these functions prove to be convex or even monotonic, even with large uncertainty intervals. For these problems, local optimisation software gives accurate results. However, theoretically, there's no guarantee that the results are even close to the exact results. Because local optimisation problems are computationally far less expensive to solve, an efficient local optimizer is the best overall choice, but the results should be examined carefully to prevent false conclusions. Furthermore, even when applying these local optimizers, the computational cost of applying direct optimisation on realistically sized transient finite element problems remains prohibitively high.

In this context, a significant reduction of the computational cost can be achieved by using response surfaces- approximations of the objective functions based on function evaluations in some well chosen points in the input parameter space. Since their introduction by Box and Wilson (1951), numerous types are developed, using different approximation functions and response points (Sacks et al 1989, Schüeller et al 1991, Montgomery 1997). Response surface based optimisation techniques prove to be extremely useful in the context of fuzzy analysis. A fuzzy analysis requires the same objective functions to be minimized and maximized on different α-levels or, in optimisation terms, with different bound constraints.

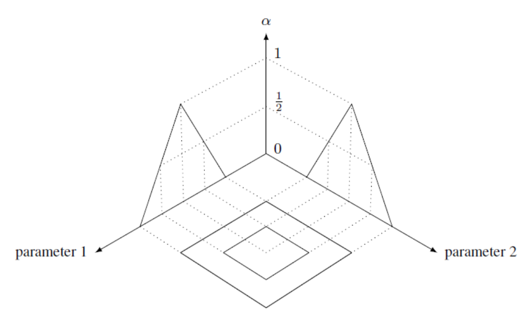

Figure 1 Optimisation bound constraints for an analysis with fuzzy uncertain parameters

Figure 1 shows this for two fuzzy uncertain parameters. The shaded rectangle shows the bound constraints for the optimisation at α-level 0.0. Response surfaces valid at this α-level should approximate the objective functions inside these bounds. The rectangles inside this shaded rectangle show the bound constraints for the optimisations at higher α-levels. It is clear that the same response surfaces approximate the objective functions at these α-levels too. Since the construction of the response surfaces is computationally by far the most expensive part of the algorithm, the cost of a fuzzy analysis is only slightly higher than that of an interval analysis when using a response surface based optimisation technique.

The objective of this study is to extend the existing interval FE methodology towards FE problems with multiple outputs. A first observation that readily can be made is that for this type of problems, the computational cost to calculate one objective function is equal to the computational cost to calculate all objective functions, since the FE solver calculates all objective functions simultaneously in each FE problem evaluation. Generic non-linear optimizers use only one of these objective functions. Other results however can be useful for the optimisations that will have to be performed on the other output quantities. For instance, results of an objective function obtained during the optimisation of another objective function can be used to select a suitable start vector. In most cases some optima, especially these located on a vertex, can be found without performing additional FE analyses. Especially for larger FE models, storing all FE analysis results can cut the computational cost significantly.

Also for multiple output FE analysis, a significant increase in efficiency can be achieved by applying the response surface methodologies as described above. It is clear that in this case, the computational cost for building the response surfaces for all objective functions is equal to the cost of building a single response surface, as long as for all objective approximations, the same reference points are used in the uncertainty space. Consequently, a multiple output interval FE analysis based on response surfaces should not be computationally much more expensive than the corresponding single output sub-problem.

An important question arises however concerning the selection of the reference sample points inside the uncertainty space. An efficient adaptive procedure to select the optimal response points is developed by the authors and is outlined below. Although here the procedure is used in combination with Kriging response surfaces (Sacks et al 1989, van Beers et al 2004), it should be noted that this approach is compatible with any response surface method which supports error estimations.

The multiple-output response surface based methodology is as follows:

1. In the first step of the procedure, a small space filling design (for example a Latin hypercube design) is generated and all objective functions are calculated at these response points by the FE solver. Using this information, initial response surfaces are created. Since these response surfaces will be improved in the second step, one should not use too many response points. The authors achieved good results with two to four times the number of uncertain parameters. Additional points are best selected by the adaptive procedure in step two instead of being randomly selected in this step.

2. In the second step, a large space filling design is calculated. These points are not yet response points; only the few most promising points from this set will become real response points for which a finite element analysis will be performed at the end of this step. For each of these points, the function value and the expected error on the function value are estimated using the calculated response surfaces. For each of these candidate response points, the average maximum improvement or AMI is calculated as

$$AMI = \sum_{\substack{k \in multi \\ outputs}} \max(\min(\tilde{f}_k(x)) - (\tilde{f}_k(x_{new}) - \Delta\tilde{f}_k(x_{new})), 0)^2 \quad (3)$$

In this formula, $\tilde{f}_k$ is an approximation of an objective function corresponding to output k of the FE analysis. The current minimum value of the approximation is $\min(\tilde{f}_k(x))$, the value of the approximation in the candidate response point is $\tilde{f}_k(x_{new})$ and $\Delta\tilde{f}_k(x_{new})$ is the error range on the approximation in this point. Thus, $\min(\tilde{f}_k(x)) - (\tilde{f}_k(x_{new}) - \Delta\tilde{f}_k(x_{new}))$ gives an indication of the improvement that can be achieved for the minimum by using this candidate response point. If the current minimum cannot be improved in this point, this value will be negative, and will be set to 0 by the $\max(x, 0)$ operation. The sum of squares selects the average maximum improvement over all output quantities. The candidate response points with the highest AMI are then selected and added to the response point set. Only in these points, an FE analysis is performed. Finally, all response surfaces are recalculated or updated with the new information. This second step of generating a large set of candidate response points and selecting the most promising points is repeated until a stopping criterion is met. One should continue the procedure until no more improvement can be made, that is, until one does not find any more points with an $AMI > 0$.

4 NUMERICAL EXAMPLE

4.1 *Description of problem*

The considered example is a model of a cable-stayed bridge, based on the pedestrian bridge over the Canal du Centre in Obourg, Belgium as illustrated in Figure 1 (Walther *et al*, 1988). The total span of this bridge is 134 m. The deck slab is made of precast prestressed concrete. The deck slab has a double T cross-section with the overall width and height being 1.8 m and 0.6 m respectively while the web thickness and flange thickness are 0.2 m and 0.3 m respectively. The pylon is A-shaped, where each arm has a rectangular cross-section of 0.8 m× 0.6 m. This pylon extends 20 m above the bridge deck and 10 m below it. The deck is supported by stranded steel cables consisting of 37 strands of 12.7 mm each.

The bridge is symmetric along the longitudinal axis. Only one half of the bridge and a single plane of cables are modelled for this analysis. The deck slab and the pylon are modelled using beam elements. The cables are modelled using bar elements. Figure 2 shows the finite element model of this bridge.

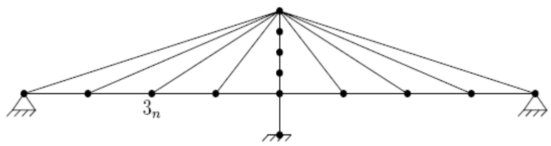

Figure 2 Simplified Finite Element Model of the Cable- stayed bridge

Two uncertain parameters are defined on the model: parameter p_1 defines the deviation of the stiffnesses of concrete and steel from their nominal value and parameter p_2 defines the deviation of the densities of concrete and steel from their nominal value. Both parameters have a maximum deviation of ±5%, resulting in fuzzy values (0.95/1.00/1.05). The coupling of the properties of concrete and steel have no physical basis; rather it is convenient to limit the number of uncertain parameters to two to be able to plot the response surfaces to illustrate the behaviour of the different methods. The bridge is subject to a sudden vertical load of 100 kN for 0.1 seconds on the bridge deck at node 3, denoted 3_n in the figure. The transient vertical displacement of node 3, resulting from this sudden vertical load, is then computed from 0 seconds to 2 seconds in 0.01 second increments (201 time steps).

4.2 *Fuzzy Analysis*

The deterministic transient vertical displacement is computed using the Wilson θ method (Wilson et al, 1972), which is an implicit integration method. To compute the displacement at a certain time, it has to be computed at all earlier time steps too. Although it is not necessary to calculate later time steps, the function is always computed at all time steps, to enable the reuse of function evaluations. The deterministic algorithm is translated to an interval algorithm using the global optimisation based approach. In this approach, the range of the displacement d3 of node 3 is determined, taking into account that the uncertain parameters p can vary within their intervals p^I. This range $\langle d_3 \rangle$ of this displacement is determined by a minimisation and a maximisation over the uncertainty interval p^I:

$$\langle d_3 \rangle = \left[\min_{p \in p^I} d_3, \max_{p \in p^I} d_3 \right] \quad (4)$$

A single interval analysis requires 402 optimisations (a minimisation and a maximisation at each time step). This interval analysis is applied at six membership levels (the support and 0.2; 0.4;…. 1.0), requiring five interval analyses at the support and at the membership levels 0.2; 0.4; 0.6 and 0.8 and one deterministic analysis at membership level 1.0. In total, 2010 optimisations and one deterministic analysis are required. The optimisations are performed using three different optimisation algorithms: mcs (a Matlab global optimisation algorithm) (Huyer & Neumaier,1999 and Neumaier, 2009), fmincon (a Matlab local optimisation algorithm) (Mathworks, 2008) and the Kriging based optimisation algorithm discussed in the previous section.

To reduce the computational cost of the optimisations using the global optimisation algorithm mcs and the local optimisation algorithm fmincon, all function evaluations of the objective function are stored in a database to enable the optimizer to reuse them for future optimisations without performing the same finite element analysis again. For the local optimisation algorithm fmincon, this database is also used to start optimisations from the point with the best function value found so far the lowest function value for minimisations and the highest function value for maximisations. All optimisations are performed from the highest to the lowest membership level.

4.3 *Results*

Figure 3 shows the upper and lower bound on the transient dynamics response of node 3 at the support, calculated using the different optimisation algorithms. From these graphs, it is clear that the global optimisation method, the local optimisation method and the Kriging based optimisation method yield the same results, except for a small difference in the lower bound around t = 1.9 seconds.

Figure 4 shows d_3, the displacement at node 3, as a function of p_1 and p_2, the two uncertain parameters, at t = 0.25 seconds (left) and at t = 1.85 seconds (right). At t = 0.25 seconds, the behaviour of d_3 is very smooth. At t = 1.85 seconds, the behaviour is less smooth, with two distinct peaks. Because the uncertain parameters influence the eigen frequencies of the bridge model, the distance between the peaks can increase or decrease, depending on the value of the uncertain parameters. In the beginning of the simulation, this causes the response to be a bit more up or down the slope, as illustrated in the left figures at t = 0.25 seconds. As the time increases, the uncertainty on the eigen frequencies causes more and more uncertainty on the exact response, the response can be on one peak or on the next peak or in the valley in between. This is clearly illustrated on the right figures at t = 1.85 seconds. If the simulation is extended beyond t = 2 seconds, even more peaks will appear in a single response.

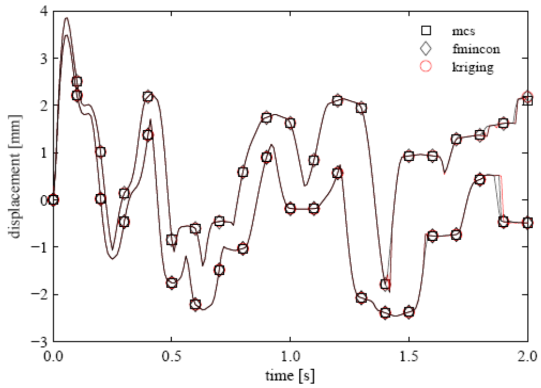

Figure 3: Bounds on the transient response at the support computed using the discussed algorithms.

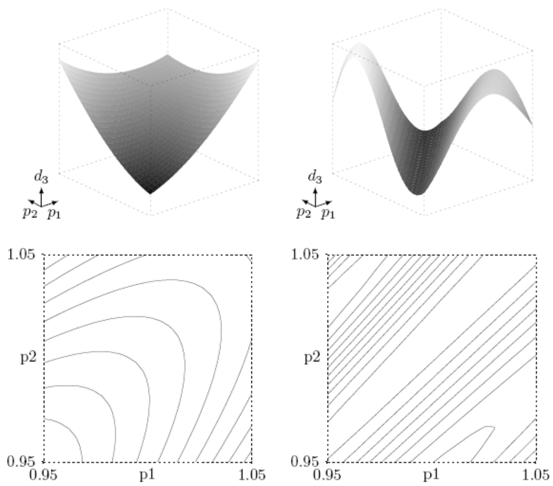

Figure 4: Surface and contour plots of the displacement at t = 0:25s (left) and at t = 1:85s (right) as a function of the uncertain model parameters p1 and p2.

Figure 5 shows the response surface created and used by the Kriging based optimisation method. These response surfaces are almost equal to the exact surfaces, as is expected based on the accuracy of the computed displacements. Figure 6 shows the fuzzy transient response of node 3, assembled from the interval responses at the support and at membership levels 0.2; 0.4; . . . ; 1.0 computed using the Kriging based optimisation method. The fuzzy responses computed using the global optimisation method and the local optimisation method are not distinguishable from the fuzzy response computed using the Kriging based optimisation method and are not reproduced here.

Until now, only the accuracy of the results is discussed. For practical applications, especially on industrially sized models, the computational cost is also very important. Table 1 shows the number of objective function evaluations (finite element analyses) and, for the response surface based methods, the number of approximation function evaluations.

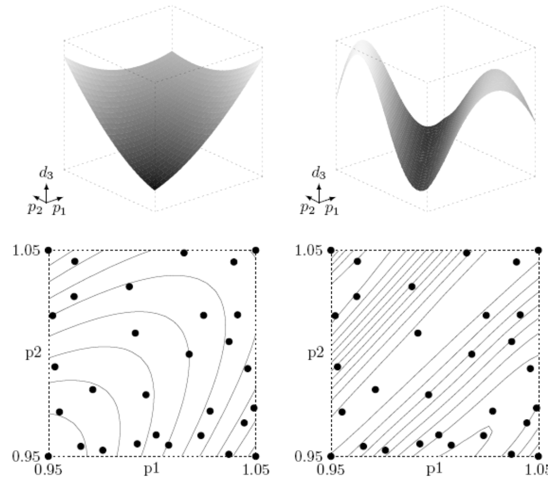

Figure 5 Surface and contour plots of Kriging approximations of the displacement at t = 0:25s (left) and at t = 1:85s (right) as a function of the uncertain model parameters p1 and p2. The response points used to construct the Kriging approximations are marked on the contour plots.

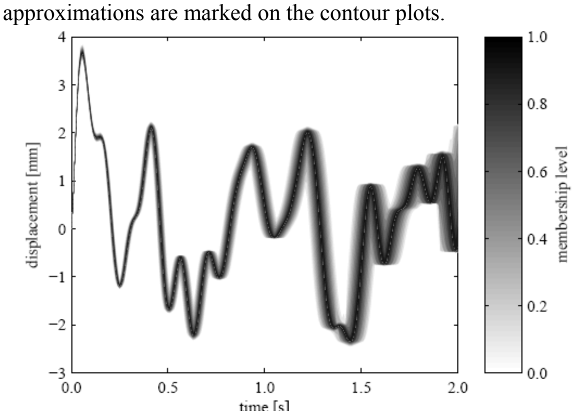

Figure 6: Fuzzy transient response computed using the Kriging based optimisation algorithm.

The computational cost of the direct optimisation methods is prohibitively high. Although the database with function values proves to be efficient, reducing the number of objective function evaluations by 75%, the global and local optimisation methods still require 47475 respectively 3363 objective function evaluations. The Kriging based method on the other hand proves to be computationally cheap, requiring only 30 function evaluations. Considering the flexibility regarding the shape of the response, the accuracy of the results and the computational cost, the Kriging based optimisation method is clearly the best choice for this problem. The cost of direct optimisation approaches is prohibitively high for industrially sized applications.

Table 1 Comparison of the computational cost of the solution of the transient dynamic finite element problem using different optimisation algorithms.

Optimisation algorithm	Function evaluations	
	Goal Function	Approximation
MCS- without database	207689	
MCS- with database	47475	
FMINCON- without database	13195	
FMINCON- with database	3363	
Kriging based algorithm	30	183815

5 CONCLUSIONS

This paper introduces a Kriging approach for the interval analysis of a transient dynamic FE problem. The method is demonstrated on a cable-stayed bridge subjected to a suddenly applied load. The method is compared to a direct optimisation approach, focussing both on accuracy and computational efficiency. It is observed that the Kriging approach is computationally far superior to the direct optimisation approach, and yields very accurate interval results. Furthermore, it was shown that the direct approach becomes problematic at higher time steps, as local optima are appearing in the displacement objective functions. The Kriging approach was shown to be effective in finding the global optima, even at these higher time steps.

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