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Analysis of a cable-stayed bridge with multiple uncertainties - A fuzzy finite element approach

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Analysis and design of structures occupy an important place in the field of Civil Engineering. In order to ensure that structures do not fail during their intended design life period with catastrophic and unpredictable consequences, proper analysis and design are mandatory. Uncertainty needs to be introduced in the engineering analysis and design of structures to enhance the functionality and dependability of the mathematical model of the structure. Classical finite element method, despite its advantages, is not suited to handle the uncertainties in the design variables of the structural system. This necessitates the development of fuzzy finite element model, which allows the use of fuzzy interval variables in order to account for multiple uncertainties associated with the structural system.

Use of fuzzy logic in structural analysis is of recent origin. Muhanna and Mullen[1] dealt with the formulation by fuzzy-finite elements for solid mechanics problems with a single uncertainty alone such as load or geometric uncertainty. Muhanna and Mullen[2] handled uncertainty in mechanics problems using an interval-based approach. Element-by-element (EBE) technique was employed to obtain a very sharp enclosure for the fuzzy solution by eliminating the sources of overestimation. Rama Rao and Ramesh Reddy[4] made the first attempt to study the structural response in the presence of multiple uncertainties of structural parameters i.e. load and Young's modulus using fuzzy finite element analysis.

Initially, a cable-stayed bridge was modelled and analysed using fuzzy-finite element analysis by Rama Rao and Ramesh Reddy[4]. Structural response of the bridge to the fuzzy interval loading was obtained in terms of fuzzy interval displacements. In the present work, the fuzzy-finite element model developed by the author[3] is used to study the structural response of a cable-stayed bridge with uncertainties in Young's modulus, live load and mass density and the results are presented.

METHODOLOGY -HANDLING MULTIPLE UNCERTAINTIES

The variational formulation for an interval case of a discrete element-by-element structural model having uncertainties associated with Young's modulus and live load is given as

$$\Pi = ½\{U\}_{\alpha\beta}^{T}[K]_{\alpha}\{U\}_{\alpha\beta} - \{U\}_{\alpha\beta}^{T}\{P\}_{\beta} + \{\lambda\}_{\alpha\beta}^{T}\{[\tilde{C}]\{U\}_{\alpha\beta} - \{0\}\} \quad (1)$$

where Π, $[K]_\alpha$, $[\tilde{C}]$, $\{U\}_{\alpha\beta}$, $\{P\}_\beta$ and $\{\lambda\}_{\alpha\beta}$ are potential energy, stiffness matrix, constraint matrix, displacement vector and live load vector and vector of Lagrange multipliers (vector of internal forces) respectively. Subscripts α and β represent the uncertainties associated with Young's modulus and live load respectively ($0\le\alpha,\beta\le1$). In this model, elements are kept separate throughout the course of the solution and constraints are imposed to ensure the compatibility of displacement of coincident nodes. Constraints are imposed on coincident nodes as

$$[\tilde{C}]\{U\}_{\alpha\beta} = \{0\} \quad (2)$$

Using Rayleigh- Ritz approach and invoking the stationarity of Π leads to

$$\{U\}_{\alpha\beta} = [\tilde{R}]^{-1}[D]_{\alpha}^{-1}\{\{P\}_{\beta} - [\tilde{C}]^{T}\{\lambda\}_{\alpha\beta}\} \quad (3)$$

where

$$[K]_{\alpha} = [D]_{\alpha}[\tilde{S}] \quad (4)$$

and

$$[\tilde{R}] = [\tilde{S}] + [\tilde{C}]^{T}[\tilde{C}] \quad (5)$$

Here $[\tilde{S}]$ is a deterministic singular matrix and $[D]_\alpha$ is a diagonal matrix containing interval terms corresponding to material uncertainty. It may here be noted that solving Eq. (3) directly leads to overestimation of the bounds of $\{U\}_{\alpha\beta}$ due to load coupling. Load coupling occurs due to:

a) Overestimation involved in the computation of external load vector $\{P\}_\beta$ owing to the superposition of contributions of various interval loads simultaneously acting on each element.
b) Overestimation involved in the subtraction of intervals in the computation of the vector $\{\{P\}_\beta - [\tilde{C}]^T\{\lambda\}_{\alpha\beta}\}$.

Thus the following procedure is adopted to avoid overestimation.

Avoiding Overestimation

The structure is analysed by *keeping the uncertain loads acting on each of the elements separate throughout the course of the solution* in order to prevent load coupling at the element level. The overall fuzzy displacement vector $\{U\}$ which represents combined effect of all joint loads and element loads is obtained by superimposing, at the end of the solution process, the fuzzy displacement vectors $\{U_{\alpha\beta}\}$ obtained for individual loads cases.

$\{\lambda\}_{\alpha\beta}$ is approximated as

$$\{\lambda\}_{\alpha\beta} \approx [\beta_l, \beta_u]\{\lambda\}_{cc} \quad (6)$$

Substituting Eq. (6) in Eq. (3) leads to

$$[D]_{\alpha}[\tilde{R}]\{U\}_{\alpha\beta} = \{\{P\}_{\beta} - [\beta_l, \beta_u][\tilde{C}]^{T}\{\lambda\}_{cc}\} \quad (7)$$

Eq. (7) can be expressed as

$$\{U\}_{\alpha\beta} = [\tilde{R}]^{-1}[M]\{\delta\}_{\alpha} \quad (8)$$

where [M] is a matrix of size n×m where n is the kinematic indeterminacy of the structure and m is the number of elements. Also, $\{\delta\}_\alpha$ is an interval vector of size m×1 containing interval Young's modulus of m elements taken from the diagonal entries of $[D]_\alpha$. Eq. (8) represents a set of linear interval matrix equations. In order to solve Eq. (8) it is necessary to compute the value of the mid-point internal force vector $\{\lambda\}_{cc}$. This is done by considering the assembled finite element model of the structure with crisp Young's modulus and fuzzy interval loading (α=1 and $0\le\beta\le.1$). The displacement vector $\{U\}_{\alpha\beta}$ is then obtained by solving Eq. (8) using Jansson's algorithm[5].

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The vector of internal forces for an element is obtained as

$$\{\lambda_{\alpha\beta}\}^{(e)}_{md\times 1} = [\beta_l, \beta_u]\,[K_\alpha]^{(e)}_{md\times md}[T]^{(e)}_{md\times md}[L]_{md\times n}[\tilde{R}]^{-1}[M]\{\delta\}_\alpha - \{P_\beta\}^{(e)}_{md\times 1} \quad (9)$$

where $[L]_{md\times n}$ is a Boolean connectivity matrix containing 0^s and 1^s, $[T]^{(e)}$ is the rotation transformation matrix for the element and md is the number of degrees of freedom for the element.

Uncertainty of mass density causes uncertainty of dead load (owing to self-weight). Thus membership function adopted for mass density is used to define the uncertainty of dead load as well. Thus load uncertainty β in Eq. (1) through Eq. (9) can be replaced by the uncertainty of mass density $\gamma\,(0\le\gamma\le1)$. The overall structural response is obtained by the superposition of structural response due to dead load and live load acting on structure.

CASE STUDY

The case study considered is a cable-stayed bridge shown in Fig. 1. The properties of the bridge are mentioned in Table 1. This problem is adopted from the configuration of *Canal du Centre Bridge* at Obourg, Belgium[6]. Owing to the symmetry of the bridge deck about the longitudinal axis of the bridge, only one half of the bridge along with a single plane of cables is used for analysis. The structural elements belonging to bridge deck and the pylon are idealized as plane frame elements while cables are modelled as bar elements. The cable-stayed bridge described above is subjected to the action of a uniformly distributed live load in addition to a uniformly distributed dead load. Linear elastic analysis is carried out and the cumulative effect of multiple uncertainties is investigated. Membership functions of Young's modulus, live load and mass density adopted for the bridge are indicated in Figs. 2 - 4, respectively. These membership functions indicate the normalised values of Young's modulus, live load and mass density respectively. An interval [a,b] can be normalized by dividing its lower and upper bounds a and b by the mid-interval value 0.5*(a+b).

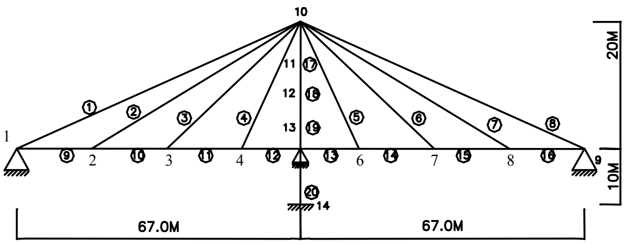

FIG. 1 CABLE STAYED BRIDGE – FAN CONFIGURATION

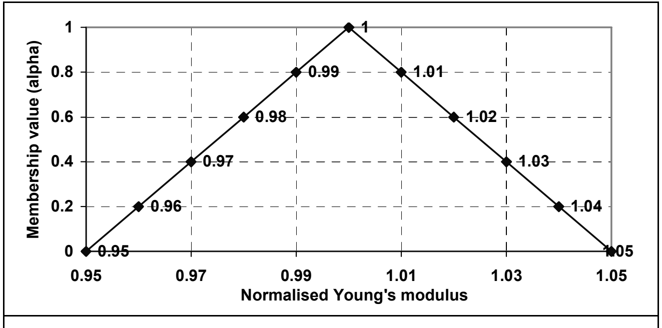

FIG. 2 MEMBERSHIP FUNCTION FOR UNCERTAINTY OF YOUNG'S MODULUS

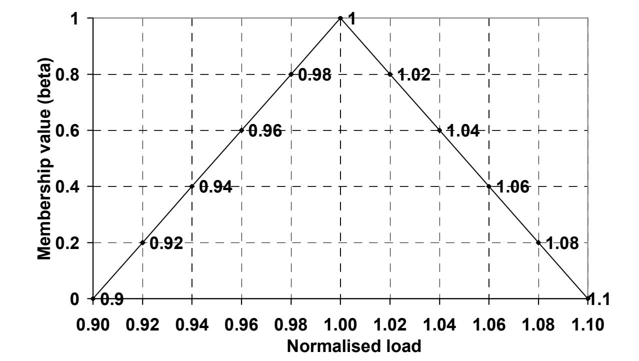

FIG. 3 MEMBERSHIP FUNCTION FOR LOAD UNCERTAINTY

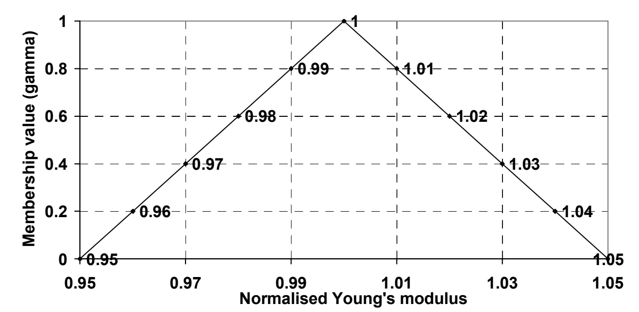

FIG. 4 MEMBERSHIP FUNCTION FOR UNCERTAINTY OF MASS DENSITY

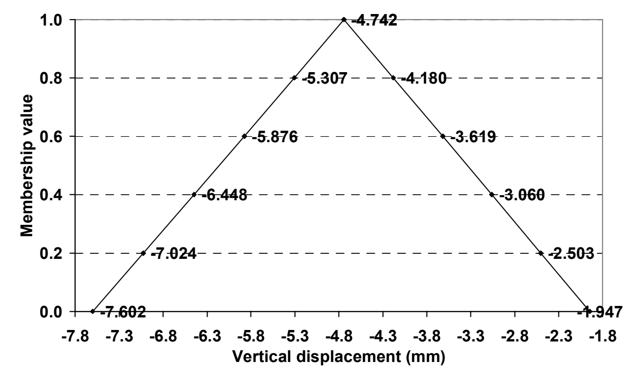

FIG. 5 COMBINED MEMBERSHIP FUNCTION FOR VERTICAL DISPLACEMENT AT NODE 3

RESULTS AND DISCUSSION

Table 2 shows the vertical displacement at node 3 for various com-

TABLE 1
PROPERTIES OF CABLE-STAYED BRIDGE

Span	134.0 m	Concrete Desk Slab		Double T Section made of Pre-cast PSC			
Overall width	1.8 m	Depth	0.6 m	Flange thickness	0.20 m	Web thickness	0.3 m
Double armed Rectangular Steel pylon, each arm 0.60×0.80 m, height 20 m above deck (30 m total height)							
Cables	Stranded steel cables each with 37 strands of 12.7 mm ϕ						
Young's Modulus	30 GPa (Concrete), 200 GPa (Steel) with uncertainty ±5% about mean for both concrete and steel						
Live Load	4.0 kN/m² with uncertainty ±10% about mean						
Mass density	2500 kg/m³(Concrete), 7850 kg/m³(Steel) with uncertainty ±5%about mean for both concrete and steel						

TABLE 2 CONCOMITANT VARIATION OF VERTICAL DISPLACEMENT OF NODE 3 ($\times10^{-2}$ m) W.R.T α AND β AT γ=0.8						
β→ α↓	1.0	0.8	0.6	0.4	0.2	0.0
1.0	[-4.885,-4.600]	[-5.015, -4.470]	[-5.145,-4.340]	[-5.275,-4.210]	[-5.405,-4.079]	[-5.535,-3.949]
0.8	[-5.177,-4.309]	[-5.307,-4.180]	[-5.438,-4.050]	[-5.568,-3.920]	[-5.699,-3.790]	[-5.829,-3.660]
0.6	[-5.470,-4.020]	[-5.601,-3.891]	[-5.732,-3.761]	[-5.863,-3.631]	[-5.994,-3.502]	[-6.125,-3.373]
0.4	[-5.765,-3.731]	[-5.897,-3.602]	[-6.028,-3.473]	[-6.160,-3.343]	[-6.291,-3.214]	[-6.423,-3.085]
0.2	[-6.061,-3.442]	[-6.193,-3.313]	[-6.325,-3.186]	[-6.457,-3.055]	[-6.590,-2.926]	[-6.722,-2.798]
0.0	[-6.359,-3.153]	[-6.491,-3.024]	[-6.624,-2.896]	[-6.757,-2.767]	[-6.889,-2.639]	[-7.022,-2.510]

TABLE 3 CONCOMITANT VARIATION OF BENDING MOMENT (kNM) IN DECK AT NODE 4 W.R.T α AND β AT γ=0.8						
β→ α↓	1.0	0.8	0.6	0.4	0.2	0.0
1.0	[-258.2,-252.6]	[-260.7,-249.9]	[-263.3,-247.3]	[-265.9,-244.7]	[-268.5,-242.1]	[-271.2,-239.5]
0.8	[-267.5,-243.3]	[-270.1,-240.7]	[-272.8,-238.1]	[-275.4,-235.5]	[-278.0,-232.9]	[-280.7,-230.3]
0.6	[-277.1,-234.1]	[-279.7,-231.4]	[-282.3,-228.9]	[-285.0,-226.3]	[-287.6,-223.8]	[-290.3,-221.2]
0.4	[-286.6,-224.9]	[-289.2,-222.3]	[-291.9,-219.8]	[-294.6,-217.3]	[-297.3,-214.7]	[-300.0,-212.2]
0.2	[-296.3,-215.7]	[-298.9,-213.2]	[-301.6,-210.6]	[-304.4,-208.1]	[-307.1,-205.6]	[-309.8,-203.0]
0.0	[-306.1,-206.6]	[-308.8,-204.1]	[-311.5,-201.6]	[-314.2,-199.1]	[-317.0,-196.5]	[-319.7,-194.0]

TABLE 4 CONCOMITANT VARIATION OF AXIAL FORCE (kN) IN CABLE 3 W.R.T α AND β AT γ=0.8						
β→ α↓	1.0	0.8	0.6	0.4	0.2	0.0
1.0	[412.3,419.2]	[409.2,422.4]	[406.0,425.5]	[402.9,428.6]	[399.8,431.8]	[396.6,434.9]
0.8	[397.1,434.8]	[394.0,438.0]	[391.0,441.2]	[387.9,444.5]	[384.8,447.7]	[381.7,450.9]
0.6	[382.2,450.8]	[379.2,454.1]	[376.1,457.4]	[373.1,460.7]	[370.1,463.9]	[367.1,467.2]
0.4	[367.4,467.2]	[364.5,470.5]	[361.5,473.9]	[358.6,477.2]	[355.6,480.5]	[352.6,483.9]
0.2	[352.9,483.8]	[350.0,487.2]	[347.1,490.6]	[344.2,494.0]	[341.3,497.5]	[338.4,500.9]
0.0	[338.7,500.8]	[335.9,504.3]	[333.0,507.8]	[330.2,511.3]	[327.4,514.8]	[324.5,518.3]

binations of α and β at γ=0.8 (±1% variation about mean value of mass density). Table 3 represents the concomitant variation of bending moment in deck at node 4 at γ=0.8 whereas Table 4 represents the variation of axial force in cable 3 at γ=0.8. It is observed that the width of the interval increases along and across in each table. Combined membership function for structural response is obtained using the α-sublevel procedure suggested by Moens and Vandepitte[7]. Figure 5 represents the combined membership function of vertical displacement at node 3. Figure 6 represents the combined membership function of bending moment at node 4. These plots represent the variation of the structural response in the presence of concomitant variation of uncertainties α ,β and γ associated with Young's modulus and live load and mass density respectively.

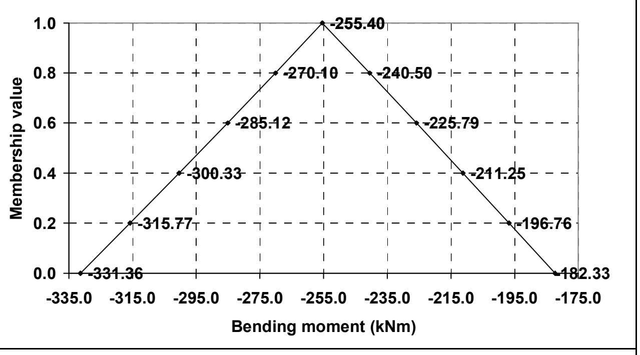

FIG. 6 COMBINED MEMBERSHIP FUNCTION FOR BENDING MOMENT AT NODE 4

CONCLUSIONS

In the present study, a new methodology to evaluate structural response in the presence of multiple uncertainties is presented. A sharp enclosure to the solution vector is obtained by uncoupling of load vector by keeping the load contributions separate throughout the solution process. The methodology is utilised to evaluate the structural response of a cable-stayed bridge in the presence of multiple uncertainties in Young's modulus, live load and mass density simultaneously. Structural response is tabulated and is found to vary for various combinations of multiple uncertainties. The effectiveness of the methodology to evaluate the structural response of a cable-stayed bridge in the presence of multiple uncertainties is demonstrated.

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