Numerical solution of FEM equations with uncertain functional parameters

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Abstract

Many numerical methods like FEM, BEM, FVM, FDM etc. leads to the problem of solution parameter dependent system of equation in the following form [1, 2]

$$A(h)u = B(h) \tag{1}$$

where A is some matrix (e.g. a stiffness matrix), B is some vector (e.g. a load vector), u is a vector of unknown (e.g. unknown nodal values) and $h = [h_1, ..., h_m]^T$ is a vector of some parameter (e.g. Yang modulus, length, volume, force etc.). In engineering practice very often we do not know the exact values of the parameters h_i. In order to create mathematical model of such systems several different method of modelling of uncertainty can be applied (e.g. probabilistic methods, imprecise probability methods, fuzzy methods, interval methods etc. [1]).

In some cases parameters h_i are not only numbers but also functions. In that case in the equation (1) we have uncertain functional parameters. One of the simplest example of such parameter is a distributed load. The solution of the equation (1) can be defined in the following way

$$u(\tilde{h}) = \{u : K(h)u = Q(h), h \in \tilde{h}\} \tag{2}$$

where $\tilde{h}$ is some functional space.

In this paper some new method of finding the solution (2) is presented. The method is based on sensitivity analysis method [1] and the concept of functional derivative [3]. Several numerical examples will be also presented.

References

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