

Finite Difference Equations with the Interval Parameters

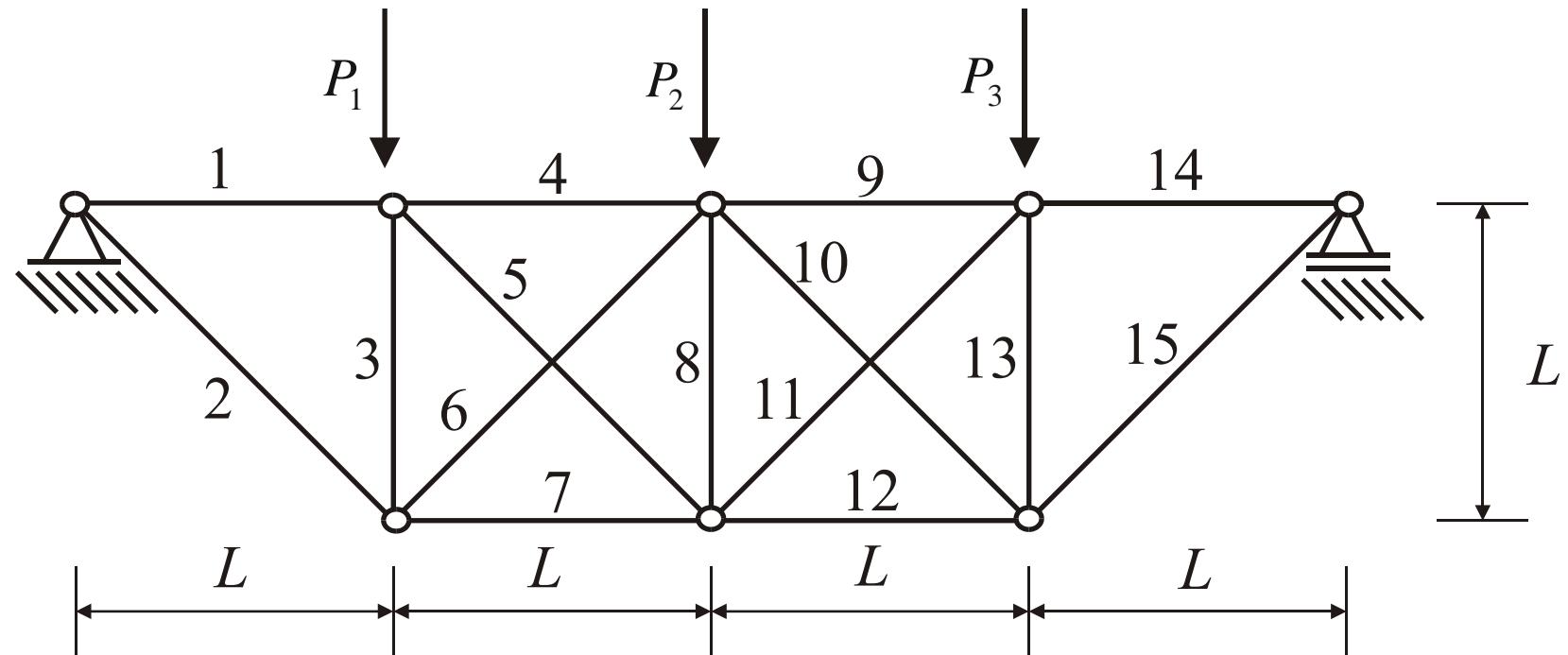
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Outline

- Modeling of uncertainty
- Set valued solutions
- Automated theorem proving

Truss structure

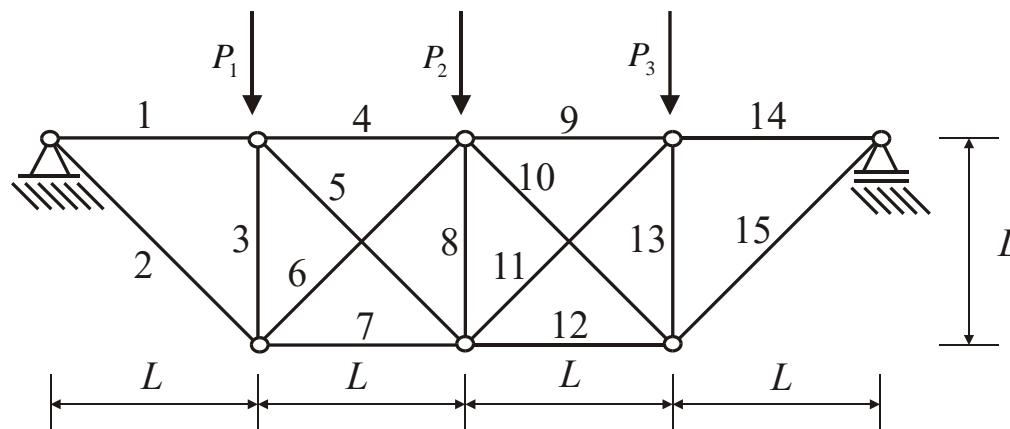


Perturbated forces

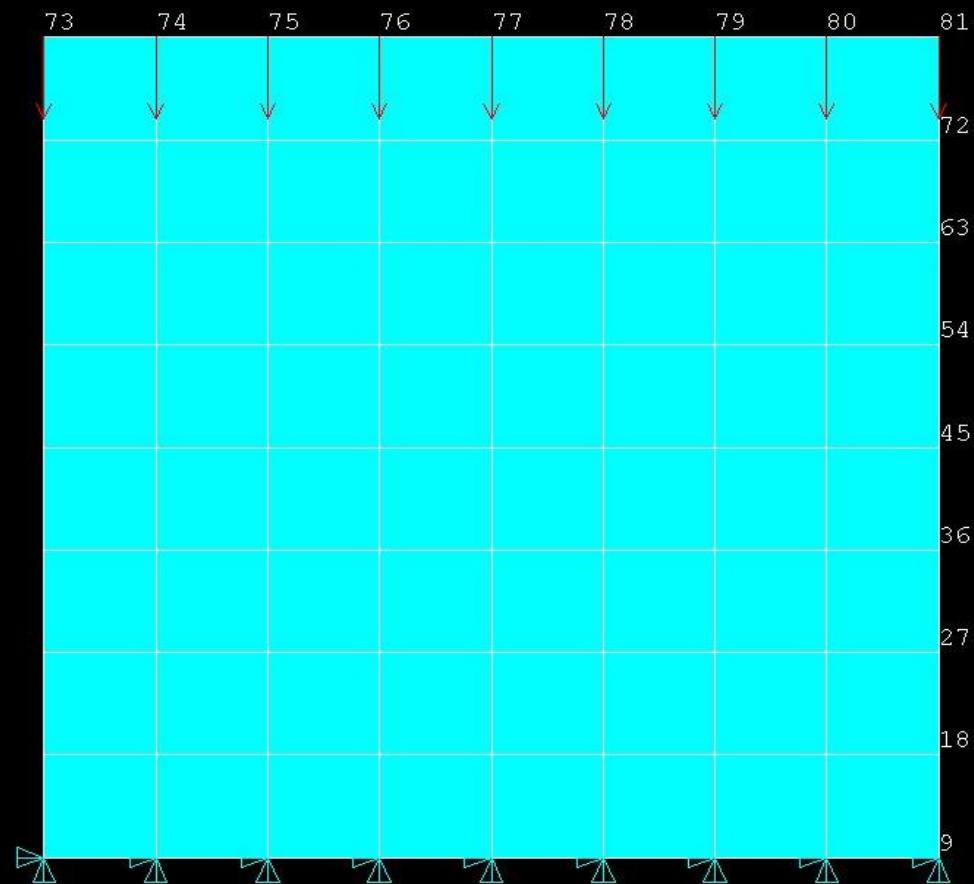
$$P = P_0 \pm \Delta P$$

5% uncertainty

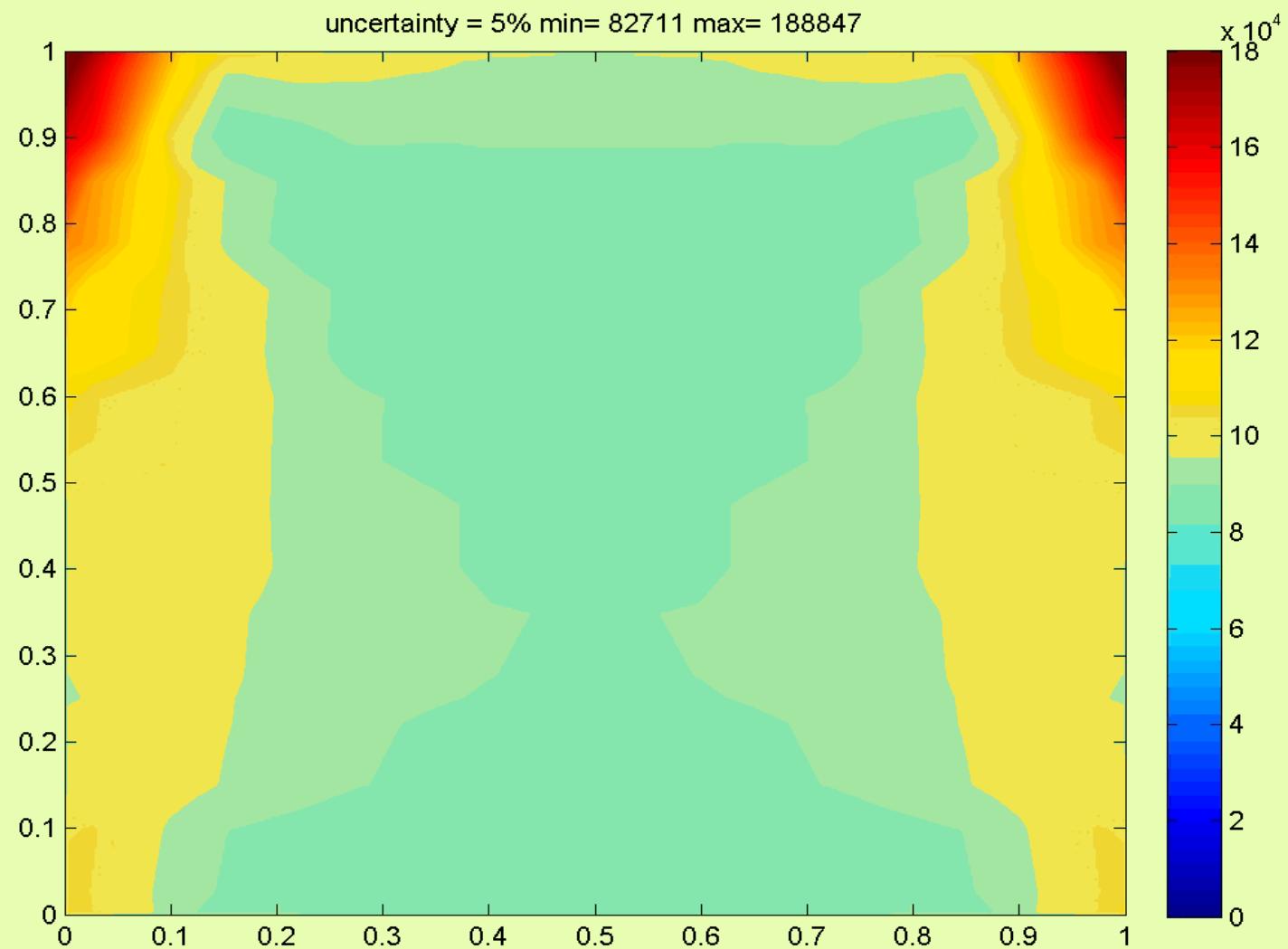
| | | | | | | | | |
|---------|----------|----------|----------|----------|----------|----------|----------|---------|
| No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| ERROR % | 10 | 9,998586 | 10,00184 | 10,00126 | 60,18381 | 11,67825 | 9,998955 | 31,8762 |
| No | 9 | 10 | 11 | 12 | 13 | 14 | 15 | |
| ERROR % | 10,00126 | 11,67825 | 60,18381 | 9,998955 | 10,00184 | 10 | 9,998586 | |



Example II (model)



Example II (results)



More examples

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Interval equations - www.pownuk.com
Pownuk A., 2000, Applications of sensitivity analysis for modelling of ...
www.pownuk.com/IntervalEquations.htm - Cache

Fuzzy sets and probability - Andrzej Pownuk home page
Fuzzy sets and probability. Scott Ferson and Lev Ginzburg, fuzzy arithmetic ...
andrzej.pownuk.com/fuzzy.htm - Cached - Similar

Web-applications related for the modeling of uncertainty
The sensitivity analysis algorithm was created by Dr. Pownuk. Theoretical ...
andrzej.pownuk.com/interval_web_applications.htm - Cached

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Friends: Jay Tea Latte, Caro Meisen, Jiseong Lee, Jungmin Kim, Volker Monty Heukens
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Andrzej M Pownuk - UTEP Faculty and Course Information
Pownuk, Andrzej M ; Trejo Marti, Juan ; Hernandez De Morales, ... Pownuk, Andrzej M . (May 2009). "Monotonicity of the solution of the interval equations of ...
hb2504.utep.edu/Profile.aspx?ID=ampownuk - Cached

User:Pownuk - Wikipedia, the free encyclopedia
Andrzej Pownuk, The University of Texas at El Paso Homepage andrzej.pownuk.com ...
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DBLP: Andrzej Pownuk
Arnold Neumaier, Andrzej Pownuk: Linear Systems with Large Uncertainties, with Applications to Truss Structures. Reliable Computing 13(2): 149-172 (2007) ...
www.informatik.uni-trier.de/~ley/db/.../a.../Pownuk:Andrzej.html - Cached

More examples

http://andrzej.pownuk.com/ Andrzej Pownuk home page

Dr. Eng. Andrzej Pownuk

[Publications] [Professional biography]
[Research interests] [Interval equations (list of references)] [Teaching] [For students] [On-line Advisor]
[Examples of applications] [Interval FEM on Wikipedia] [Programming Experience]
[Computational Science Ph.D. Program at UTEP] [Tools]
[My schedule] [CPS]

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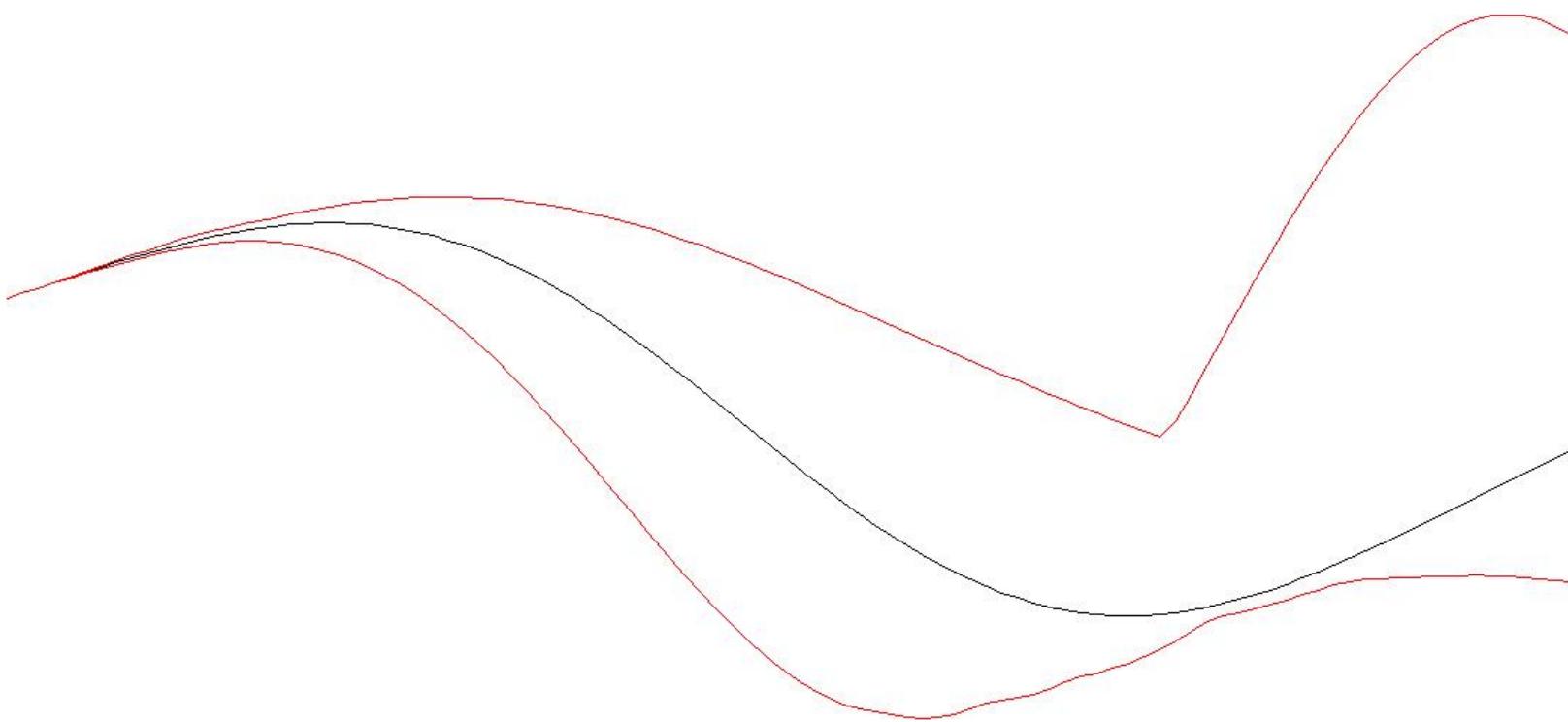
El Paso, 2007

Second order approximaiton

$$u'' + (k/m)u = (P/m)\cos(wt)$$

$$v = u'$$

Calculation

 $u_0 = 1$
 $v_0 = 1$ $P = 1$ $k_{\min} = 1$ $k_{\max} = 2$ $m_{\min} = 1$ $m_{\max} = 2$ $w = 2$ number of intermediate points = 5 $dt = 0.05$ number of timesteps = 100

E = dE = % min E = 190000000000 max E = 210000000000 rho0 = [kg/m³]
 A = dA = % min A = 0.0095 max A = 0.0105 drho = %
 J = dJ = % min J = 7.91635E-06 max J = 8.74965E-06 min rho = 7480.3
 dt = [s] P = [N] Time steps for load = max rho = 8267.7
 L = [m] n = Total time when the load was applied = 0.001 [s]
 Ln = [m]

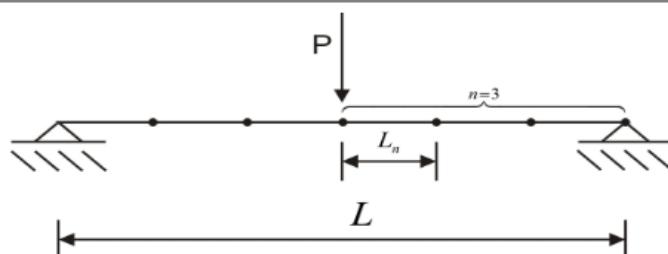
Number of interval parameters: 8

List of nodes

Number of timesteps = 600

node 1, x = 0
 node 2, x = 5
 node 3, x = 10

Number of DOF = 9



number of elements = 2

Dof in nodes

Number of nodes = 3

- node 0
- 0 1 2
- node 1
- 3 4 5
- node 2
- 6 7 8

Nodes in elements

- element 0
- 0 1
- element 1
- 1 2

DOF in elements

- element 0
- 0 1 2
- 3 4 5
- element 1
- 3 4 5
- 6 7 8

Newmark method's parameters:

beta =

gamma =

Solution to calculate, node = DOF =

Stifness matrix

Mass matrix

Show local stifness matrix

Show local mass matrix

Stiffness matrix with PC

Mass matrix with PC

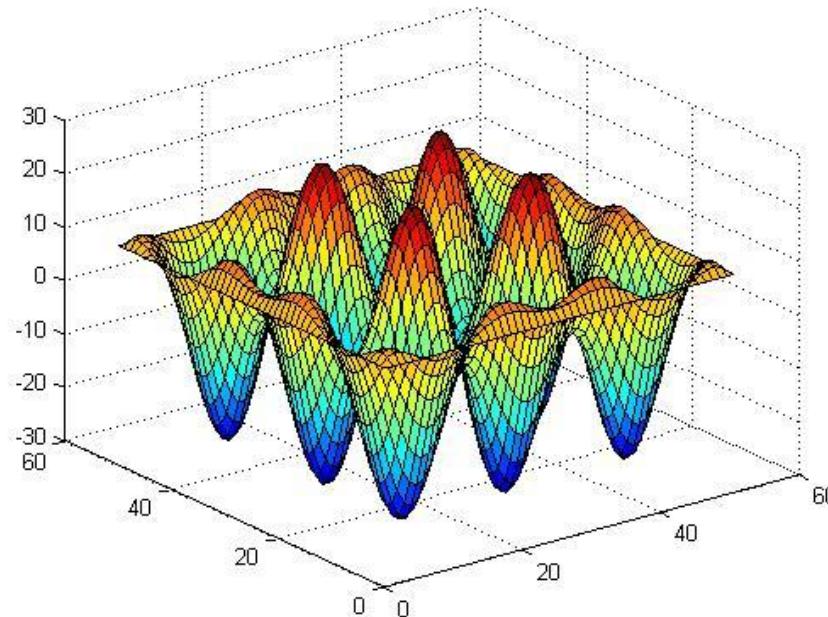
PC

PC

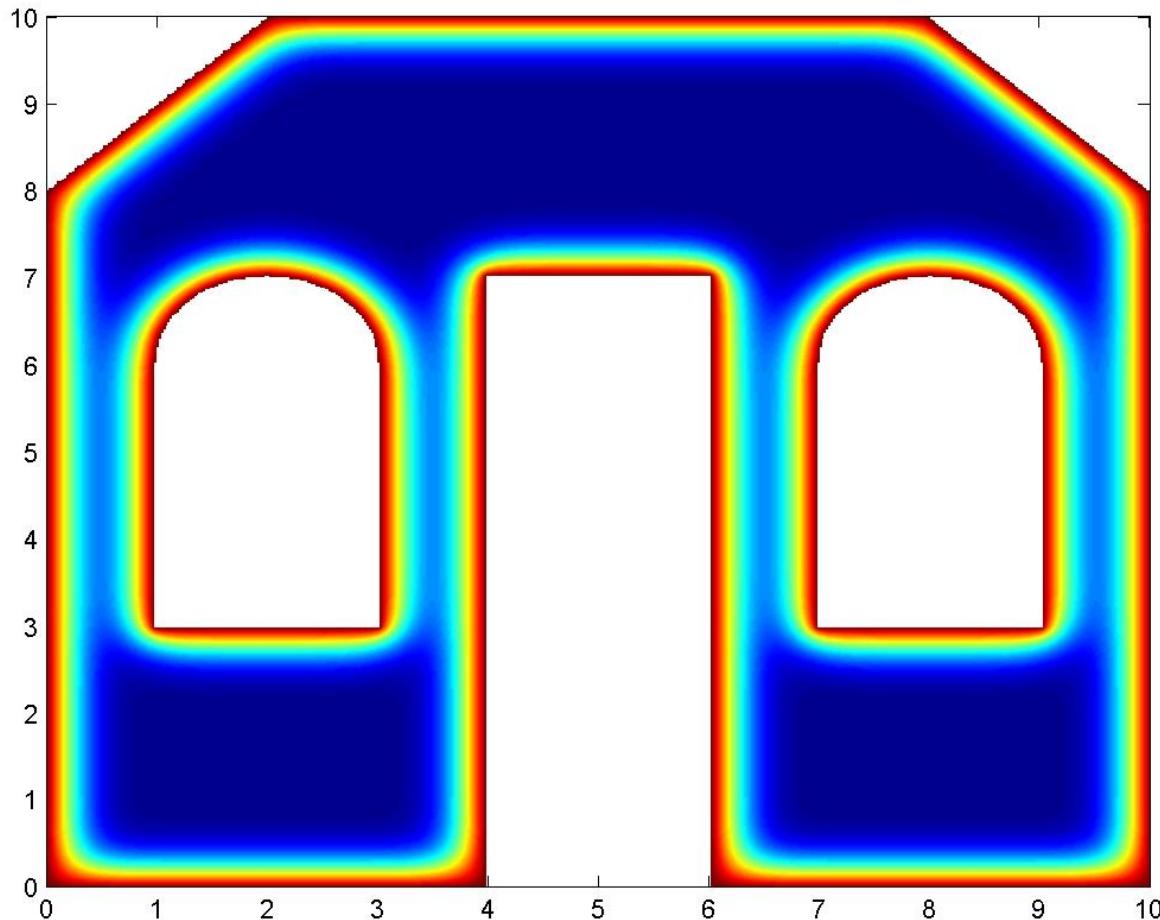
2D PDE

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + T = f$$

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} + T_{i,j} = f_{i,j}$$



Solution of 2D heat equation

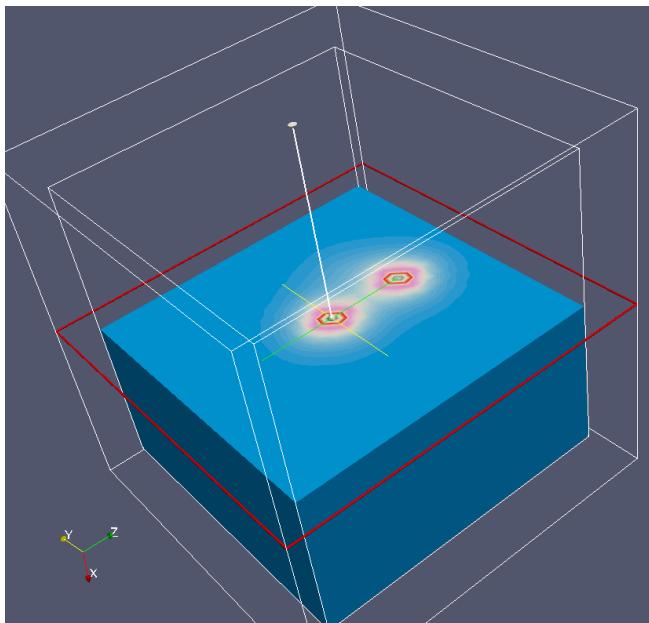


3D Heat transfer problem

$$\alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q = \frac{\partial T}{\partial t}$$

$$\alpha \left(\frac{T_{i+1,j,k}^t - 2T_{i,j,k}^t + T_{i-1,j,k}^t}{\Delta x^2} + \frac{T_{i,j+1,k}^t - 2T_{i,j,k}^t + T_{i,j-1,k}^t}{\Delta y^2} + \frac{T_{i,j,k+1}^t - 2T_{i,j,k}^t + T_{i,j,k-1}^t}{\Delta z^2} \right) + q_{i,j,k} = \frac{T_{i,j,k}^{t+\Delta t} - T_{i,j,k}^t}{\Delta t}$$

$$T_{i,j,k}^{t+\Delta t} = T_{i,j,k}^t + \alpha \left(\frac{T_{i+1,j,k}^t - 2T_{i,j,k}^t + T_{i-1,j,k}^t}{\Delta x^2} + \frac{T_{i,j+1,k}^t - 2T_{i,j,k}^t + T_{i,j-1,k}^t}{\Delta y^2} + \frac{T_{i,j,k+1}^t - 2T_{i,j,k}^t + T_{i,j,k-1}^t}{\Delta z^2} \right) \Delta t + q_{i,j,k} \Delta t$$



Engineering simulations



Uncertain parameters:

$$ax = b \quad x = \frac{b}{a}$$

Example: $[1, 2]x = [1, 4]$

$$x = \frac{[1, 4]}{[1, 2]} = [1, 2]$$

Extreme values of monotone functions

$$f(x) = x^2, \quad x \in [1, 2]$$

$$\frac{df(x)}{dx} = 2x \in [2, 4]$$

$$\underline{f} = f(\underline{x}) = f(1) = 1^2 = 1$$

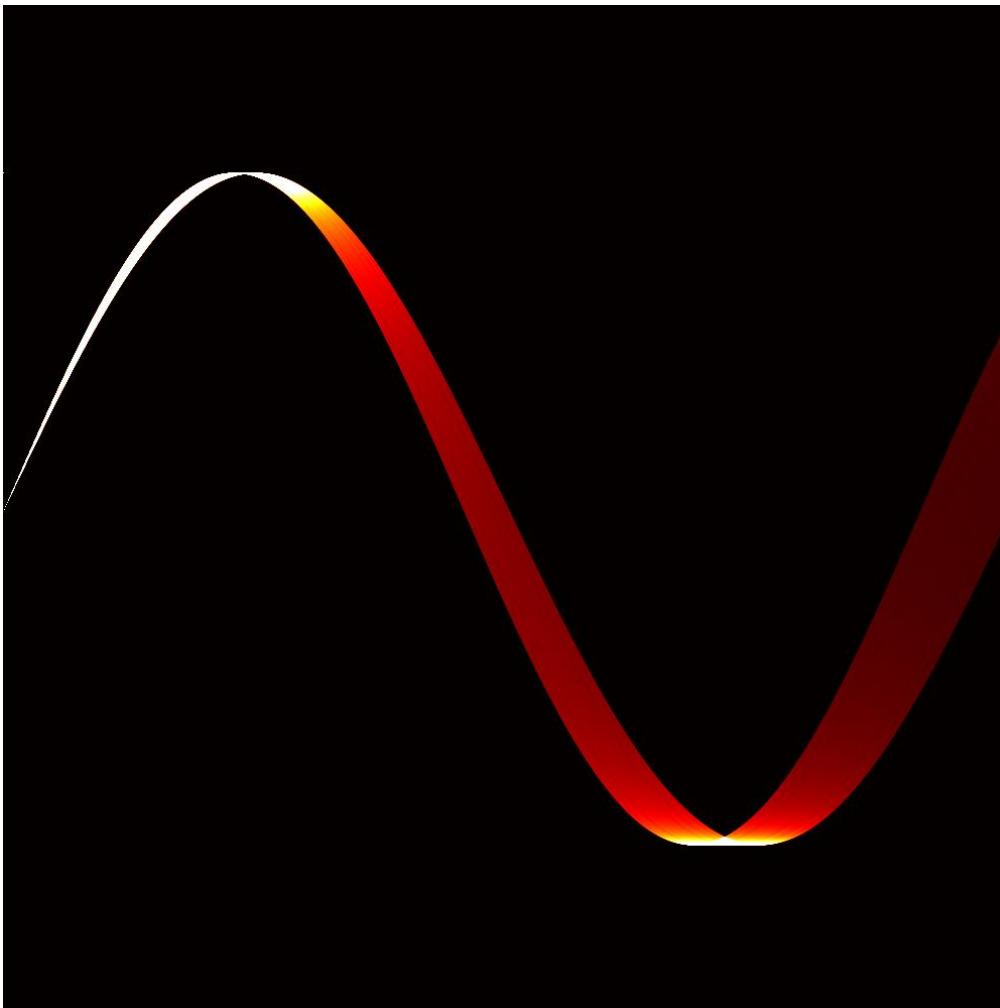
$$\bar{f} = f(\bar{x}) = f(2) = 2^2 = 4$$

$$f(x) \in [\underline{f}, \bar{f}] = [1, 4]$$

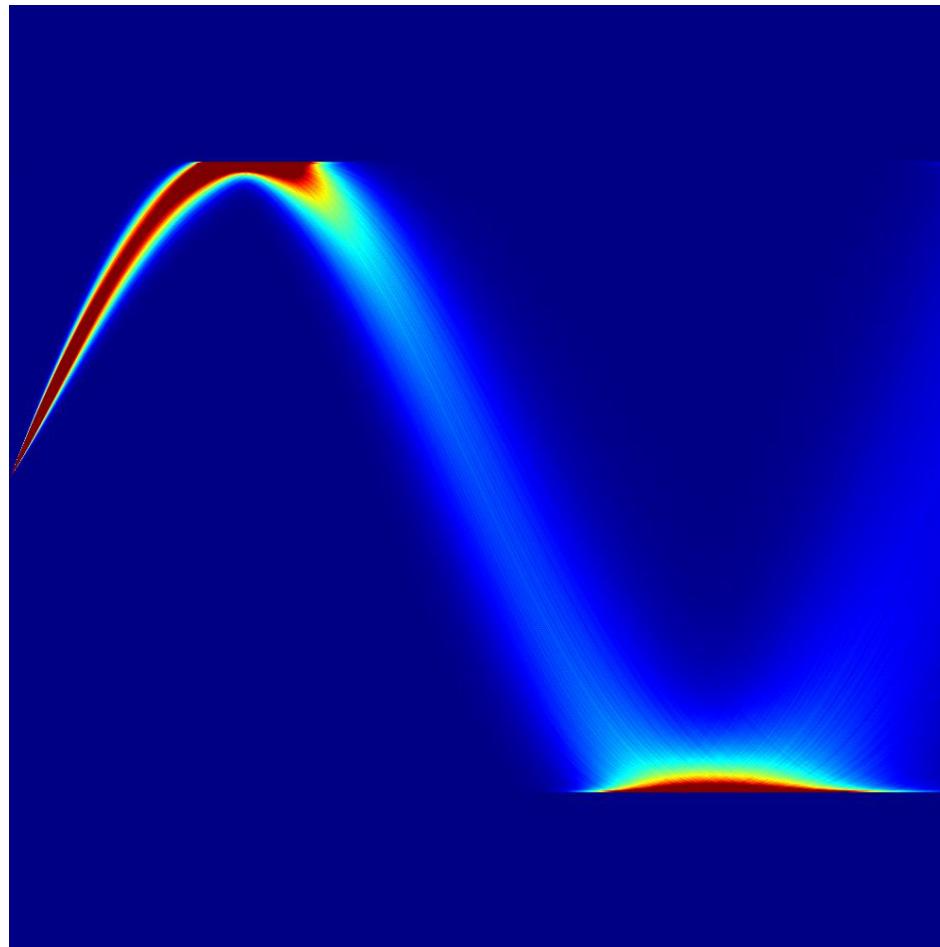
Interval solution of PDE

$$u(x,p) = \{u(x,p) : p \in \mathbf{P}\}$$

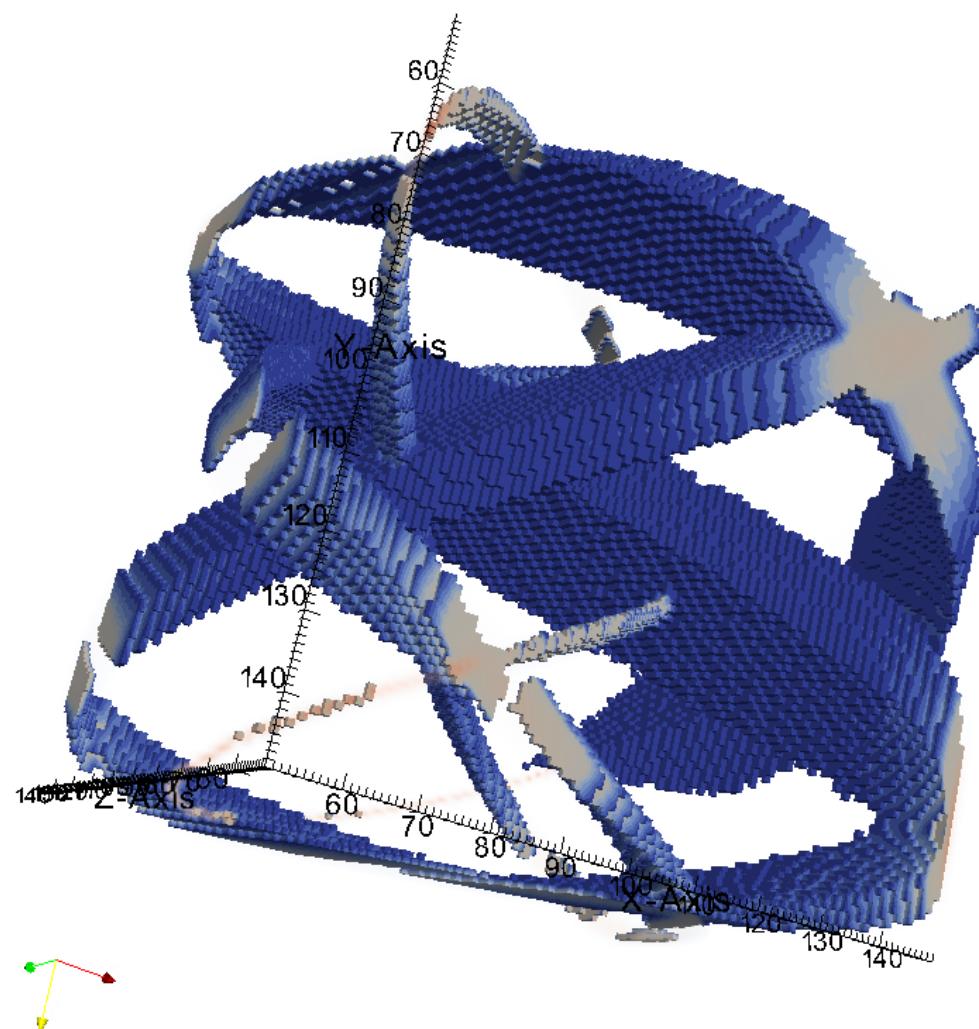
Set valued solution of ODE



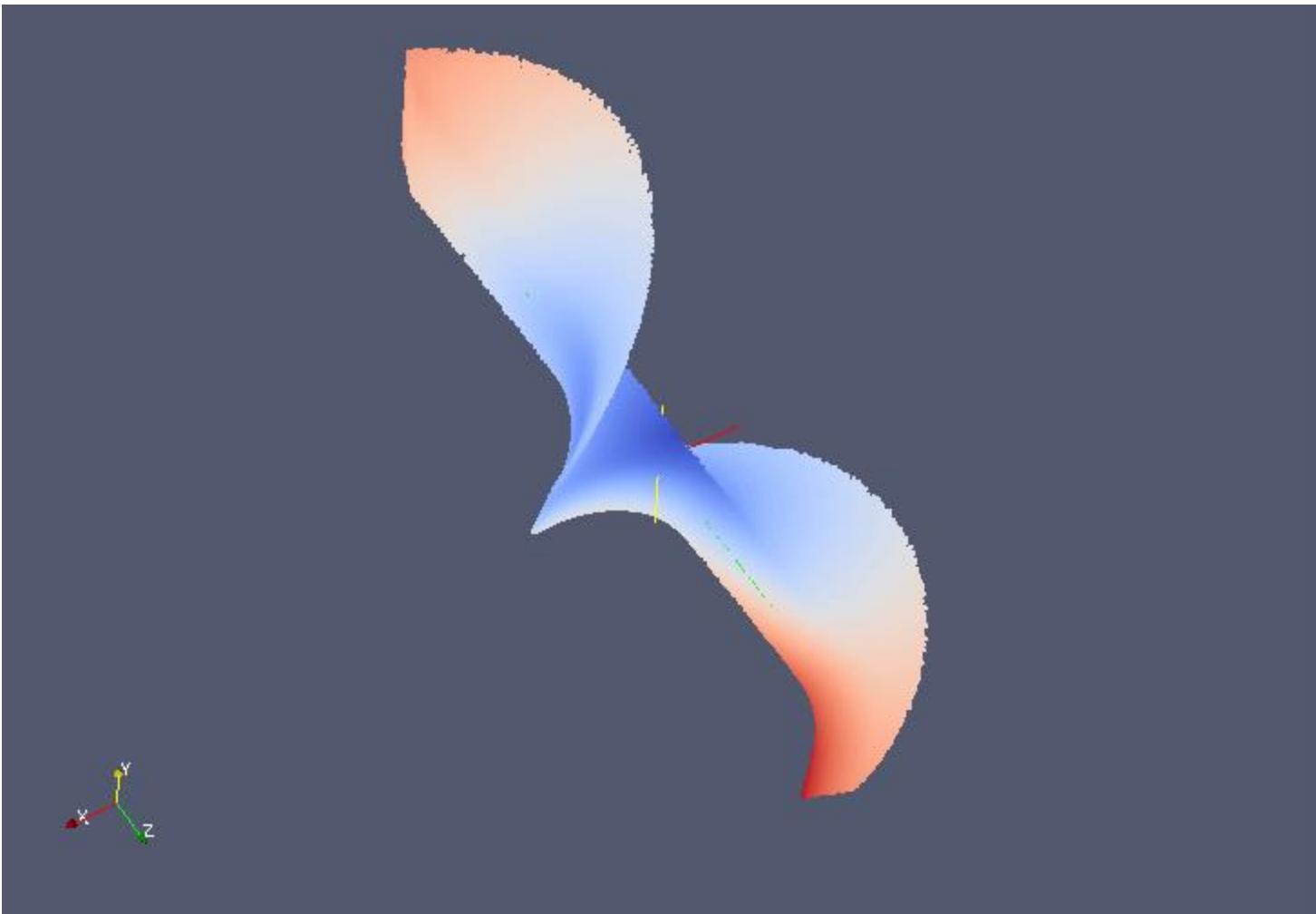
Random solution of ODE



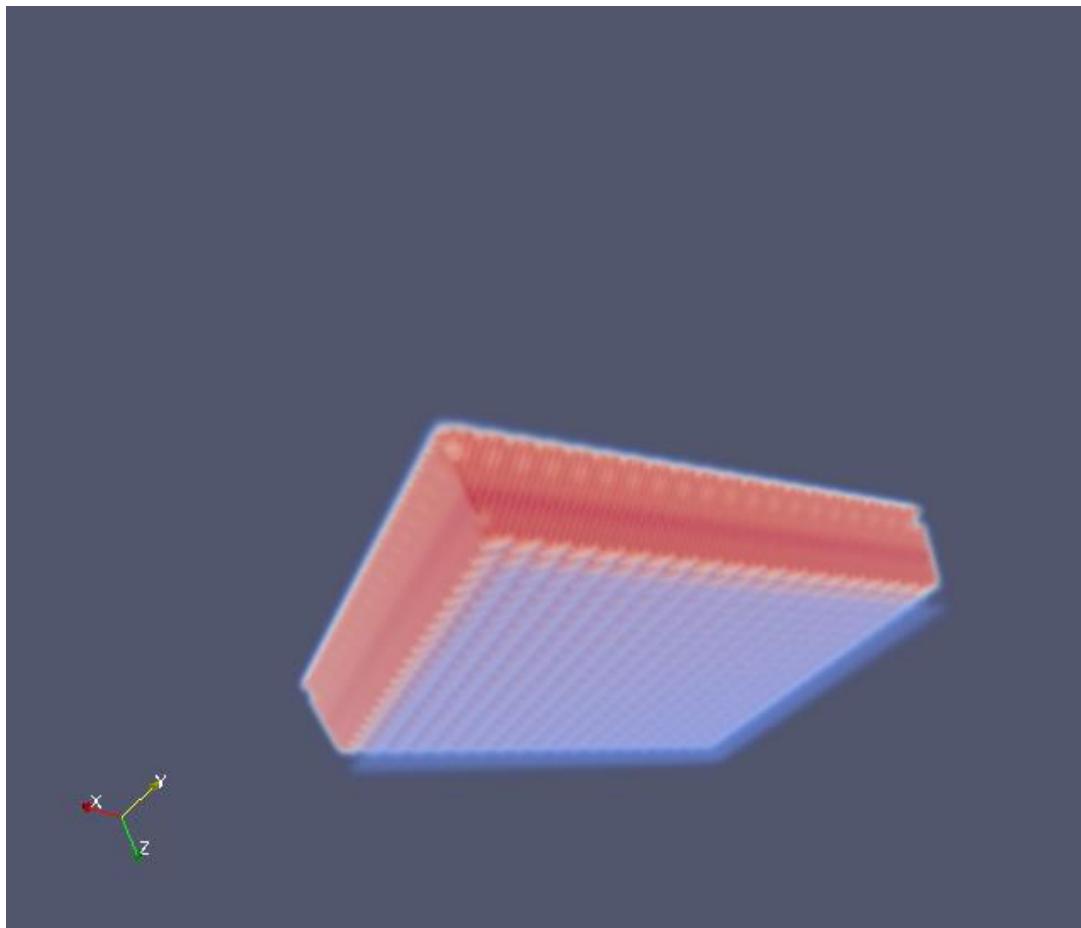
Set valued solution of ODE



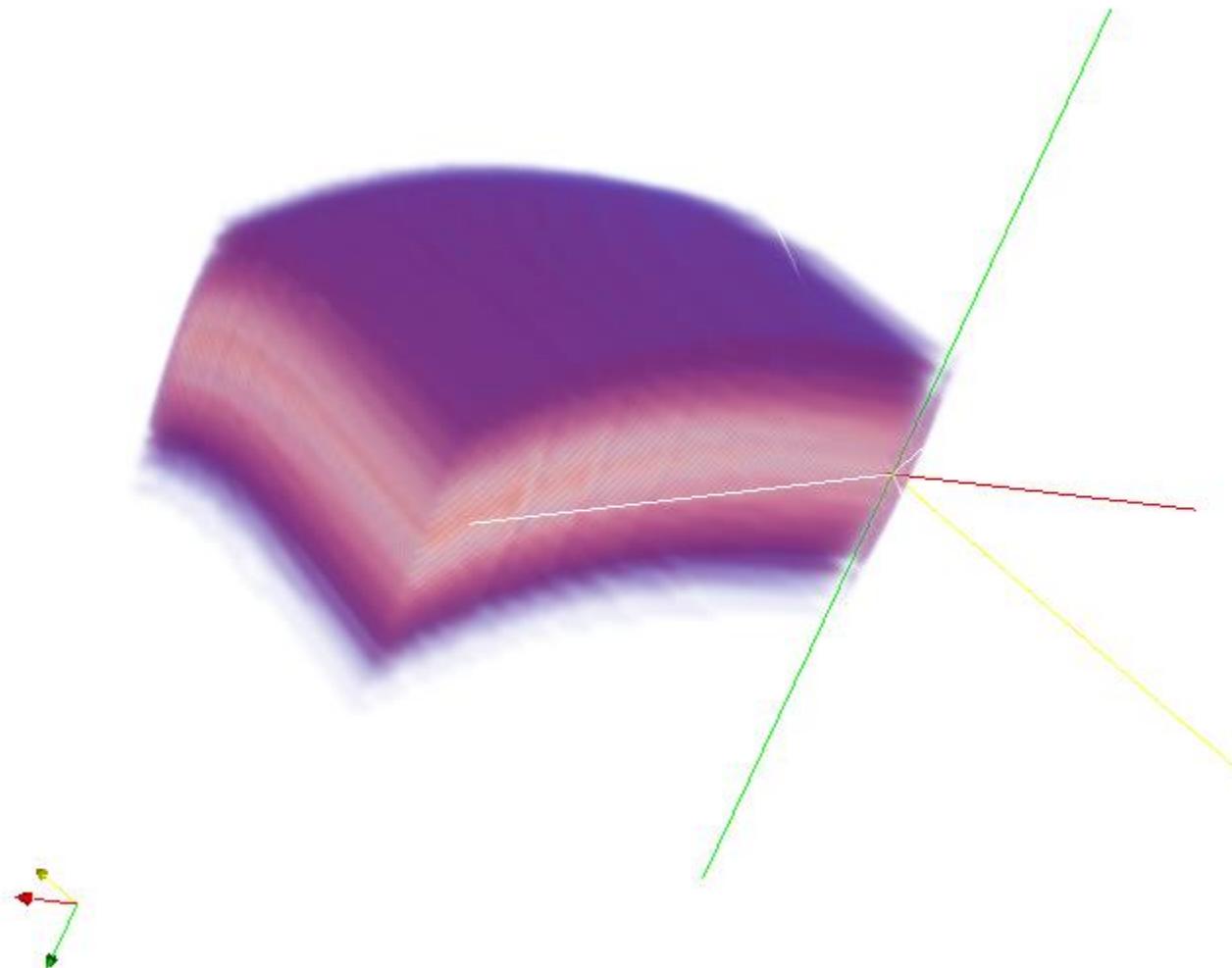
Solution set of algebraic equations



Set valued solution of PDE



Set valued solution of PDE



- Solution set of the equations with the interval parameters is not an interval
 - it is not necessary to reduce the set-valued solution the the interval solution.

$$\mathbf{u}(\mathbf{p}) = \square u(\mathbf{p})$$

- it is possible to use the set-valued solution directly in calculations.

Finite difference method

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f$$

$$\left(\frac{\partial^2 u}{\partial x^2} \right)_{ij} + \left(\frac{\partial^2 u}{\partial y^2} \right)_{ij} = f_{ij}$$

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} = f_{ij}$$

$$A(p)u = b(p)$$

Gradient method

$$A(p) \frac{\partial u}{\partial p} = \frac{\partial b(p)}{\partial p} - \frac{\partial A(p)}{\partial p} u$$

$$\underline{u}_i = \begin{cases} \text{min } u_i \\ A(p)u = b(p), \quad p \in \mathbf{P} \end{cases} \quad \bar{u}_i = \begin{cases} \text{max } u_i \\ A(p)u = b(p) \\ p \in \mathbf{P} \end{cases}$$

Optimization

$$p_i^{\text{in}} = \arg \min \left\{ \begin{array}{l} \text{m in } u_i \\ A(p)u = b(p) \\ p \in \mathbf{P} \end{array} \right.$$

$$p_i^{\text{max}} = \arg \max \left\{ \begin{array}{l} \text{m ax } u_i \\ A(p)u = b(p) \\ p \in \mathbf{P} \end{array} \right.$$

New solution procedure

Stokes theorem

$$\int_{\Sigma} \nabla \times \mathbf{F} \cdot d\Sigma = \oint_{\partial\Sigma} \mathbf{F} \cdot d\mathbf{r},$$

Butterfly lemma or Zassenhaus lemma

Lemma: Suppose (G, Ω) is a group with operators and A and C are subgroups. Suppose

$B \triangleleft A$ and $D \triangleleft C$

are stable subgroups. Then,

$(A \cap C)B / (A \cap D)B$ is isomorphic to $(A \cap C)D / (B \cap C)D$.

**It is possible to use
mathematical theorems in an
automatic/computational way
without human presence.
(automated theorem proving)**



It is possible to apply automatic theorem proving to the solution of the interval equations.

Automated theorem proving
can be applied to the solution
any scientific field.
(biology, chemistry, physics ...)

however this is not the topic of my research
look for example at: www.cyc.com
theory of everything ...



It is possible to build a system
who is able to use its own
results in order to improve
itself (self learning).

Examples

- $2*[1,2]*x - x = 1$
- $(2*[1,2] - 1)x = 1$
- $([2,4] - 1)x = 1$
- $[1,3]x = 1$
- $x = 1/[1,3]$
- The system is able to show all intermediate steps of the calculations

Look also at:

<http://www.wolframalpha.com>

Example

$$\frac{du}{dx} + up = 0, u(0) = 0, p \in [1, 2]$$

$$\frac{u_{i+1} - u_i}{\Delta x} + u_i p = 0, u_0 = 0, p \in [1, 2]$$

$$u_{i+1} = u_i p \Delta x + u_i, u_0 = 0, p \in [1, 2]$$

$$\underline{u}_i = \min \left\{ u_i : u_{i+1} = u_i p \Delta x + u_i, u_0 = 0, p \in [1, 2] \right\}$$

$$\bar{u}_i = \max \left\{ u_i : u_{i+1} = u_i p \Delta x + u_i, u_0 = 0, p \in [1, 2] \right\}$$

Example

- cats like food
 - food=[milk< canned food<mice]
(example problem with the interval data)
-
- What kind of food do the cats like most?
 - Answer: mice



Conclusions

- It is possible to solve efficiently the finite difference equations of computational mechanics with the interval parameters.
- In many cases it is possible to use general solution set of the system of interval equations instead of replacing it by the interval solution set.
- It is possible to apply automatic theorem proving in order to solve many problems in the theory of interval equations.
- It is possible to create self improving computational systems.