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Evaluation of Safety of Structures with Fuzzy Parameters

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Abstract

This paper presents a method for evaluating the safety of structures with fuzzy input parameters. Uncertainty in load and material properties is defined by triangular membership functions with equal spread about the crisp value. The structure is modeled using the fuzzy-finite element approach developed earlier by the first author. The structural response is obtained in terms of fuzzy interval displacements and rotations. The results are further post-processed to obtain interval values of bending moment, shear force and axial force. Combined membership functions for uncertain forces in the elements are obtained using the α-sublevel approach using this interval output. Fuzzy membership functions of stresses in the elements are evaluated and are compared with the corresponding membership functions for the allowable stresses. Safety of the structures is evaluated at various levels of uncertainty of the input parameters. The present work demonstrates the effectiveness of uncertainty based analysis and design in establishing reliability of structural design for a structure with uncertain input parameters.

INTRODUCTION

Analysis and design of structures play a vital role in the field of Civil Engineering. Proper analysis and design are mandatory in order to ensure that structures meet their intended design life period without catastrophic and unpredictable consequences. Modelling with intervals provides a link between design and analysis where uncertainty may be represented by bounded sets of parameters. The aim of the designer is to predict the entire spectrum of behaviour by mathematical formulation and identification of those parameters which influence this behaviour and to obtain the cracking, deflection and collapse limit loads Thus the design process becomes clear only when the process of analysis is learnt thoroughly.. In the Limit State Design method, the structure shall be designed to withstand safely all loads likely to act on it throughout its life. For ensuring the design objectives, the design should be based on characteristic values for material strengths and applied loads (actions), which take into account the probability of variations in the material strengths and in the loads to be supported. The design values are derived from the characteristic values through the use of partial safety factors, both for material strengths and for loads (IS 800-1984). Partial factors of safety specified for load and yield stress are usually 1.5 and 1.15 respectively. The reliability of design is ensured by requiring that design action is less than the design strength throughout the projected life of the structure. Thus the code of practice allows for an additional increase of load by 50 percent and reduction of yield stress by 15 percent. However these limits to the variation of load and material properties may not be realistic. The code of practice does not allow one to foresee the effect of a larger variation of these values on the safety of the structures designed using the provisions of the national codes. This mandates the introduction of uncertainty into analysis and design process.

A reliable approach to handle uncertainty in a structural system is the use of interval algebra [Muhanna and Mullen (1999)]. In this approach, uncertainties in structural parameters are introduced as interval values i.e., the values are known to lie between two limits, but the exact values are unknown. Thus the functionality and dependability of the mathematical model of the structure are enhanced.

ANALYSIS AND DESGIN OF STRUCTURES WITH INTERVAL PARAMETERS

In this paper a method for evaluating the safety of structures with fuzzy input parameters in presented. Uncertainty in load and material properties is defined by triangular membership functions with equal spread about the mean value. The structure is modeled using the fuzzy-finite element approach developed earlier by the first author [Rama Rao and Ramesh Reddy (2006), Rama Rao and Ramesh Reddy (2007)]. Uncertainties of Young's modulus, load and cross sectional area are considered. Multiple uncertainties are incorporated into the fuzzy-finite element model of the structure using an element-by-element approach with Lagrange multipliers and the methodology is applied to the analysis of several structural problems.

The structural response is obtained in terms of fuzzy interval displacements and rotations. The results are further post-processed to obtain interval values of bending moment, shear force and axial forces. Combined membership functions for uncertain forces in the elements are obtained using the α-sublevel approach [Moens and Vandepitte (2005)]. The fuzzy membership functions of stresses in the elements are evaluated and are compared with the corresponding membership functions for the allowable stresses. The safety of the structures is evaluated at various levels of uncertainty of input parameters.

FUZZY FINITE ELEMENT MODELLING OF STRUCTURES

The variational formulation for an interval case of a discrete element-by-element structural model having uncertainties associated with Young's modulus and live load is given as[Rama Rao and Ramesh Reddy (2006), Rama Rao and Ramesh Reddy (2007)]

$$\Pi = \frac{1}{2}\left\{U_{\alpha\beta}^{T}\right\}\left[K_{\alpha}\right]\left\{U_{\alpha\beta}\right\} - \left\{U_{\alpha\beta}^{T}\right\}\left\{P_{\beta}\right\} + \left\{\lambda_{\alpha\beta}^{T}\right\}\left\{\left[\tilde{C}\right]\left\{U_{\alpha\beta}\right\} - \{0\}\right\} \quad (1)$$

where $\Pi, \left[K_{\alpha}\right], \left[\tilde{C}\right], \left\{U_{\alpha\beta}\right\}, \left\{P_{\beta}\right\} and \left\{\lambda_{\alpha\beta}\right\}$ are potential energy, stiffness matrix, constraint matrix, displacement vector and load vector and vector of Lagrange multipliers respectively. Subscripts α and β represent the uncertainties associated with Young's modulus and load respectively. The range of α and β is 0 to 1. In this model, elements are kept separate throughout the course of the solution and constraints are imposed to ensure the compatibility of the displacement of coincident nodes. Constraints are imposed on coincident nodes as

$$\left[\tilde{C}\right]\left\{U_{\alpha\beta}\right\} = \{0\} \quad (2)$$

Using the Rayleigh- Ritz approach and invoking the stationarity of Π leads to

$$\left\{U_{\alpha\beta}\right\} = \left[\tilde{R}^{-1}\right]\left[D_{\alpha}^{-1}\right] - \left\{\left\{P_{\beta}\right\} - \left[\tilde{C}\right]^{T}\left\{\lambda_{\alpha\beta}^{T}\right\}\right\} \quad (3)$$

where

$$\left[K_{\alpha}\right] = \left[D_{\alpha}\right]\left[\tilde{S}\right] \text{ and } \left[\tilde{R}\right] = \left[\tilde{S}\right] + \left[\tilde{C}\right]^{T}\left[\tilde{C}\right] \quad (4)$$

Here $\left[\tilde{S}\right]$ is a deterministic singular matrix and $\left[D_{\alpha}\right]$ is a diagonal matrix containing interval terms corresponding to material uncertainty at specified α level. The structure is analysed by keeping the uncertain loads acting on each of the elements separate throughout the course of the solution in order to prevent load coupling at the element level. Displacement vector $\left\{U_{\alpha\beta}\right\}$ is obtained by solving Eq. (4) using Jansson's algorithm (Jansson,1991). The vector of internal forces for an element is obtained as

$$\left\{\lambda_{\alpha\beta}\right\} = \left[\beta_{l}, \beta_{u}\right]\left[K_{\alpha}\right]\left[K\right]\left[L\right]\left[\tilde{R}^{-1}\right]\left[M\right]\left\{\delta_{\alpha}\right\} - \left\{P_{\beta}\right\} \quad (5)$$

where $[L]$ is a Boolean connectivity matrix and $[T]$ is the rotation transformation matrix for the element.

TENSION AND COMPRESSION OF STRUCTURES WITH INTERVAL PARAMETERS

Three example problems are considered in the present work i.e. a stepped bar subjected to axial tension, a fifteen element truss structure and a portal frame. These structures are subjected to fuzzy loading and have fuzzy geometric and material properties. These structures are modelled using the fuzzy finite element approach developed by the first author, and the structural response is obtained at various levels of membership value ranging from 0 through 1. Combined membership functions for the axial stresses in elements are plotted using the α-sublevel approach [Moens and Vandepitte (2005) and Buckley (1990)]. The detailed procedure for ascertaining the safety of these structures is outlined in the following sections.

Bar and Truss Structures with Interval Parameters

The first example considered is a stepped bar subjected to axial tension P as shown in Fig.1. Young's modulus E is 200 GPa. Areas of cross section are $A_1 = 1.2\times10^{-3}$ m^2, $A_2 = 0.7\times10^{-3}$ m^2. Lengths are L_1=1m, L_2=0.5m and applied loads $P_1$and P_2 are 80 kN each. The uncertainty is expressed as a percentage variation about the mean value of a parameter. The uncertainty of cross sectional area is ±5 percent and load uncertainty is ±20 percent. The corresponding triangular fuzzy membership functions are shown in Fig. 2 and Fig.3. Consider the stresses σ_1 and σ_2, and the allowable stress in axial tension $\sigma_{t,allow}$ as

$$\sigma_1 = \frac{P}{A_1}\ ,\ \sigma_2 = \frac{P}{A_2}\ \text{and}\ \sigma_{t,allow} = 0.6 f_y \tag{6}$$

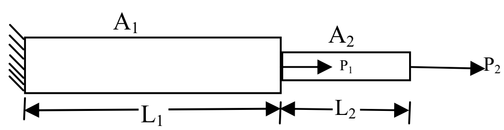

Fig. 1 Stepped Bar under Axial Tension

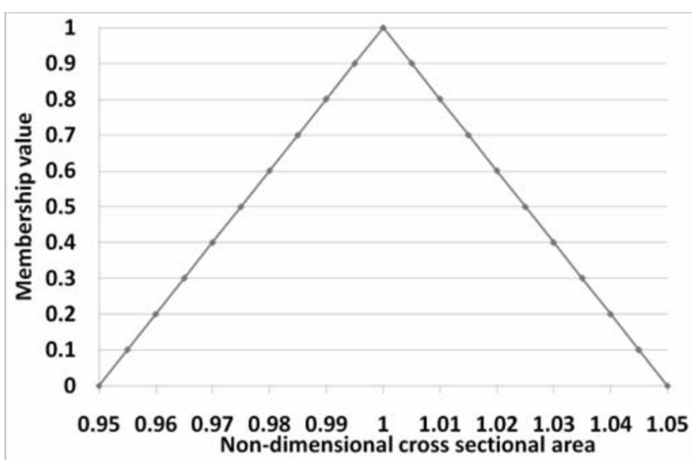

Fig.2 Stepped Bar-Membership Function for Coss Sectional Area

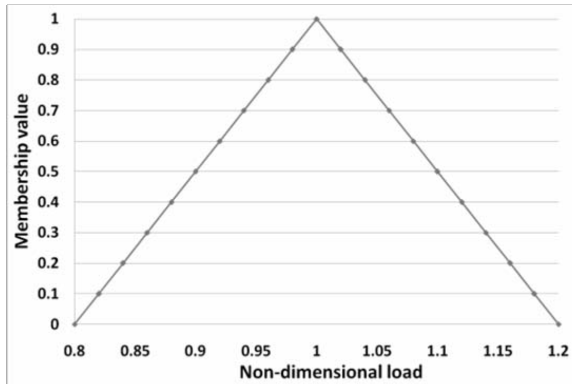

Fig.3Stepped Bar-Membership Function for Load

For mild steel, the nominal value of yield stress f_y is $250\mathrm{MPa}$. However, uncertainty may be associated with yield stress owing to variations in the manufacturing process of steel. Thus, a maximum variation of ±5percent is considered in the yield stress of steel. As per IS 800-1984, the structure is safe if

$$\sigma_1, \sigma_2 \leq \sigma_{t,allow} \tag{7}$$

Fig. 4 shows the membership function for the allowable stress $\sigma_{t,allow}$. Table 1 presents the fuzzy interval values of σ_1, σ_2 and $\sigma_{t,allow}$ at various levels of $\alpha\ (0 \leq \alpha \leq 1)$. Fig.5 presents the fuzzy membership functions of $\sigma_{t,allow} - \sigma_1$ and $\sigma_{t,allow} - \sigma_2$.

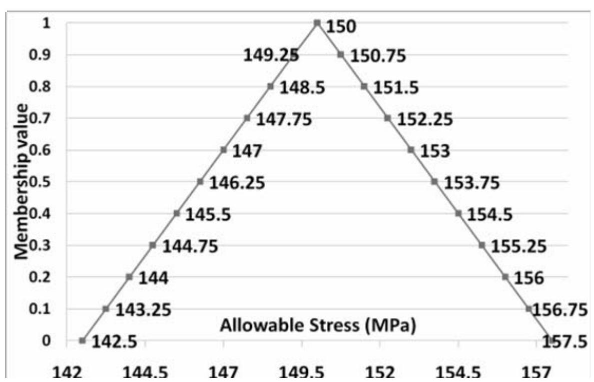

Fig.4 MembershipFunction for Allowable Stress

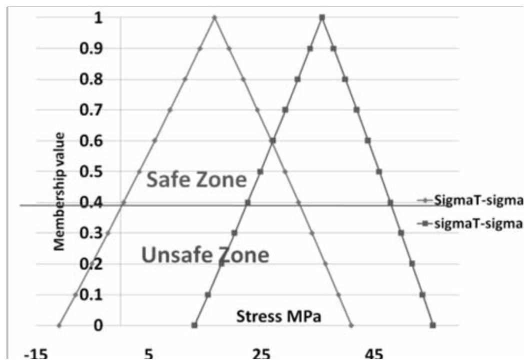

Fig.5 Stepped Bar-Safe Bounds for Stress

Table 1 Stresses in Stepped Bar (MPa)

α	σ_1	σ_2	$\sigma_{t,allow}$
1	[133.333, 133.333]	[114.286, 114.286]	[150.00, 150.00]
0.8	[126.733, 140.067]	[108.628, 120.058]	[148.50, 151.50]
0.6	[120.261, 146.939]	[103.081, 125.948]	[147.00, 153.00]
0.4	[113.916, 153.952]	[97.642, 131.959]	[145.50, 154.50]
0.2	[107.692, 161.111]	[92.308, 138.095]	[144.00, 156.00]
0	[101.587, 168.421]	[87.075, 144.361]	[142.50, 157.50]

A thick horizontal line is drawn at the level α_S where these membership functions cut the y-axis. The region defined by $\alpha_S \le \alpha \le 1$ represents the safe zone because $\sigma_{t,allow} - \sigma_1$ and $\sigma_{t,allow} - \sigma_2$ are positive in this zone. The region defined by $0 \le \alpha < \alpha_S$ represents the unsafe zone. In the present example, $\alpha_S \approx 0.4$. This corresponds to an acceptable variation of ±36 percent for the load and ±3 percent for the cross sectional area. Any variations beyond these limits are unacceptable in view of ensuring structural safety.

Eleven-Element Truss Structure with Fuzzy Parameters

Fig. 6 shows a two-bay truss with 11 elements with fuzzy interval value of Young's modulus and subjected to fuzzy interval loading. The cross sectional area of each element of the truss is 6.84×10^{-4} m^2 and Young's modulus is 200GPa. A load of 150 kN acts at the joint 2.

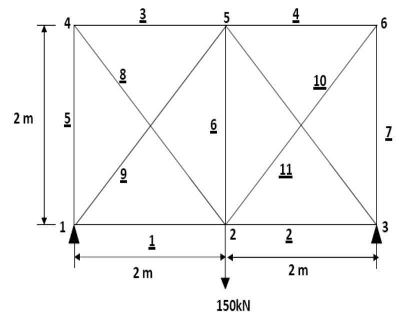

Fig. 6 Truss Structure with 11 Elements

Uncertainties of load and Young's modulus adopted for the truss are ±60 percent and ±5 percent respectively. Fuzzy triangular membership functions can be plotted to represent these uncertainties. Normalised interval values of load and Young's modulus are extracted from these membership functions at each level of α and are used in the computations of interval values of displacements and forces. The corresponding values of axial stress at each α level are obtained by dividing the interval axial force with the corresponding area of cross section of the element. The combined membership function for the axial stress is plotted using the α -sublevel approach.

Fig. 7 shows the combined membership function for the tensile stress in element 6 (the element carrying the maximum stress in the truss structure) obtained using this procedure. The allowable stress for the material in direct tension is given as $\sigma_{t,allow} = 0.6 f_y$ where f_y is the yield stress of the material. Thus, a maximum variation of ±5percent is considered in the yield stress of steel in this case as well.

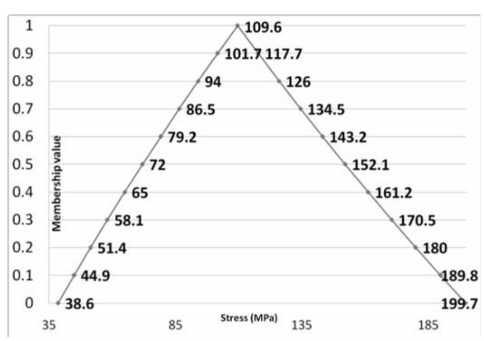

Fig.7 Tensile Stress in Element 6

Considering σ and $\sigma_{t,allow}$ are fuzzy numbers, we must have

$$\sigma < \sigma_{t,allow} \tag{8}$$

from consideration of structural safety. Fig. 8 shows the membership function for $(\sigma_{t,allow} - \sigma)$.

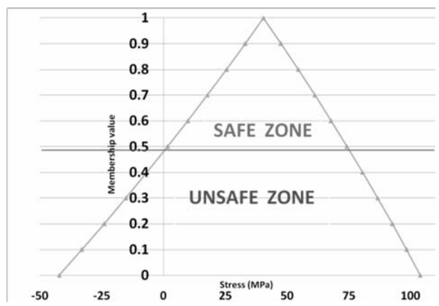

Fig.8 Acceptable Stress Level in Element 6

It is observed from this membership function that $(\sigma_{t,allow} - \sigma) \geq 0$ for the zone $(0.5 \leq \alpha \leq 1.0)$ indicating a safe zone and $(\sigma_{t,allow} - \sigma) < 0$ for the zone $(0 \leq \alpha < 0.5)$ indicating a unsafe zone. Thus the corresponding bounds of load and Young's modulus for the safe zone are [0.7,1.3] and [0.975,1.025]. These values indicate that a maximum variation of 30 percent in load and 2.5 percent is Young's modulus are allowed from consideration of structural safety.

Framed Structure with Interval Parameters

In the case of framed structures the maximum stress can be calculated as a combination of bending and axial stresses

$$\sigma = \sigma_d \pm \sigma_b \tag{9}$$

Where direct stress $\sigma_d = \dfrac{N}{A}$ and bending stress $\sigma_b = \dfrac{M}{Z}$ with M as its bending moment and N as the axial force, A as the area of cross-section and Z is the section modulus. In the simplest case Z can be calculated as

$$Z = \frac{J}{z_{\max}} \tag{10}$$

As per the Indian steel code (IS 800-1984), the structure is safe if

$$\sigma_t \leq \sigma_{t,allow} \tag{11}$$

in tension and

$$\left|\sigma_c\right| \le \sigma_{c,allow} \tag{12}$$

in compression and

$$\sigma_b \le \sigma_{b,allow} \tag{13}$$

in bending, subject to the additional condition

$$\frac{\sigma_t}{\sigma_{t,allow}} + \frac{\sigma_b}{\sigma_{b,allow}} \le 1 \tag{14}$$

for combined axial tension and bending and

$$\frac{\sigma_c}{\sigma_{c,allow}} + \frac{\sigma_b}{\sigma_{b,allow}} \le 1 \tag{15}$$

for combined axial compression and bending. Here $\sigma_{t,allow} = 0.6 f_y$, and $\sigma_{b,allow} = 0.66 f_y$, $\sigma_{c,allow} = \min(0.6 f_y, \sigma_{ac})$ where

$$\sigma_{ac} = 0.6 f_y \frac{f_{cc} f_y}{\left[f_{cc}^n + f_y^n\right]^{\frac{1}{n}}} \quad , \; f_{cc} = \frac{\Pi^2 E}{\lambda^2} \tag{16}$$

with n = 1.4 , E is the Young's modulus, λ is the slenderness ratio of the member and f_y is the yield stress.

A single bay- single storey portal frame as shown in Fig.9 is chosen as an illustrative example. The columns have a cross section of 0.3m×0.2m and the beam has a cross section of 0.3m×0.25m. Young's modulus is 200GPa. Load P = 150kN and q = 37.5kN/m. The membership function of Young's modulus adopted for the truss problem is adopted in this case as well. Load uncertainty adopted is ±100 percent. The structural response of the portal frame is obtained at various levels of membership value ranging from 0 through 1 using the procedure described earlier. Interval values of axial force and bending moment are computed using this procedure. The corresponding values of axial stress and bending stress and combined stress are computed and are used to validate Eq.(14). The computed values are presented in Table 2.

It is observed from Table 2 that interval combined stresses for membership values α corresponding to the range 0.3<α≤1.0 satisfy Eq. (14), implying that the structure is safe in this range. Beyond this, the structure is unsafe in the range 0≤α≤0.3 as Eq. (14) is violated. Fig. 10 shows a plot of combined stresses σ_1 and σ_2 . In this figure, the membership values above the thick horizontal line at $\alpha = 0.3$ represent the limit interval stresses satisfying Eq. (14). A similar analysis for safety of members subjected to combined axial compression and bending can be performed using Eq. (15).

Table 2 Interval Forces and Moments in Element 1

α	Axial Force (kN)	Bending Moment (kNm)	$\frac{\sigma_t}{\sigma_{t,allow}}+\frac{\sigma_b}{\sigma_{b,allow}}$
0	[0,185.9]	[0, 26.45]	[0.000,1.219]
0.1	[7.5,174.7]	[10.4, 245.3]	[0.049,1.144]
0.2	[15.3,163.7]	[21.0, 229.5]	[0.098,1.071]
0.3	[23.2,152.9]	[31.9, 214.1]	[0.149,0.999]
0.4	[31.3,142.3]	[43.1, 199.1]	[0.201,0.929]
0.5	[39.5,132.0]	[54.5, 184.4]	[0.255,0.860]
0.6	[47.9,121.9]	[66.2, 170.0]	[0.309,0.793]
0.7	[56.5,111.9]	[78.2, 156.0]	[0.365,0.728]
0.8	[65.3,102.2]	[90.4, 142.3]	[0.422,0.664]
0.9	[74.3,92.7]	[102.9, 128.8]	[0.480,0.601]
1.0	[83.4,83.4]	[115.7, 115.7]	[0.540,0.540]

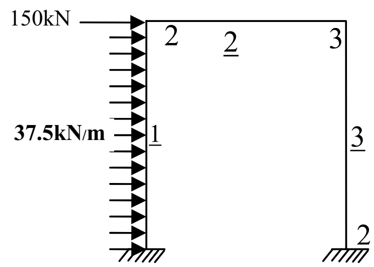

Fig.9 Portal Frame

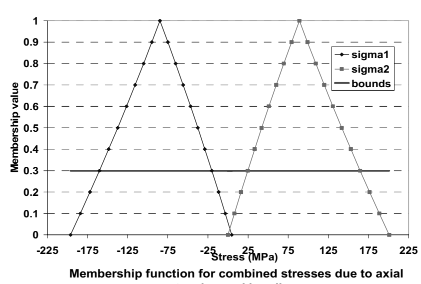

Fig.10 Membership Function For Combined Stresses

CONCLUSIONS

In the present work, structures with uncertain structural parameters are modeled using fuzzy finite element method. Numerical results obtained in this paper indicate that it is possible to evaluate the safety of bar, truss and frame structures with the interval and fuzzy parameters. The present approach can be extended to the design of two-dimensional or three-dimensional structures with a given set of limit state equations.

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