Application of Regular Interval Jacobian Matrices to Calculation of Extreme Values of Mechanical Quantities

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Many mechanical systems can be described by the parameter dependent system of equation *F(z,t)=0* ($F : R^n \times R^m \to R^n$) [1]. In this paper, a new method for calculating the range of functions $z_i(t)$ over the box $\mathbf{T}$ is presented.

$$\underline{z}_i = inf\left\{z_i(t) : t \in \mathbf{T}\right\}, \bar{z}_i = sup\left\{z_i(t) : t \in \mathbf{T}\right\} \tag{1}$$

Other methods are described in [2-4]. In many engineering problems functions $z_i(t)$ are monotone in $\mathbf{T}$. In this case we can calculate extreme values using vertices of multidimensional interval $\mathbf{T}$

$$\underline{z}_i = min\left\{z_i(t) : t \in V\right\}, \bar{z}_i = max\left\{z_i(t) : t \in V\right\} \tag{2}$$

where V is a set of vertices of interval $\mathbf{T} \in IR^m$. In this paper I proved that, if the following interval Jacobi matrices

$$\frac{\partial F(\mathbf{Z},\mathbf{T})}{\partial z}, \frac{\partial F(\mathbf{Z},\mathbf{T})}{\partial(z_1,...,z_{i-1},t_j,z_{i+1},...,z_n)} \tag{3}$$

are regular ($\mathbf{Z} = \mathop{\times}\limits_{i=0}^{n}[\underline{z}_i, \bar{z}_i]$), then all implicit functions $z_i(t)$ are monotone in the box $\mathbf{T}$ and we can calculate extreme values (1) using formulas (2). Using implicit functions theorem [4], we can calculate partial derivatives of $z_i(t)$

$$\frac{\partial z_i(x)}{\partial t_j} = -\left|\frac{\partial F(x)}{\partial(z_1,...,z_{i-1},t_j,z_{i+1},...,z_n)}\right| \cdot \left|\frac{\partial F(x)}{\partial z}\right|^{-1} \tag{4}$$

My theorem is conclusion from equation (4).

References

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