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Applications of sensitivity analysis for modelling of structures with uncertain parameters

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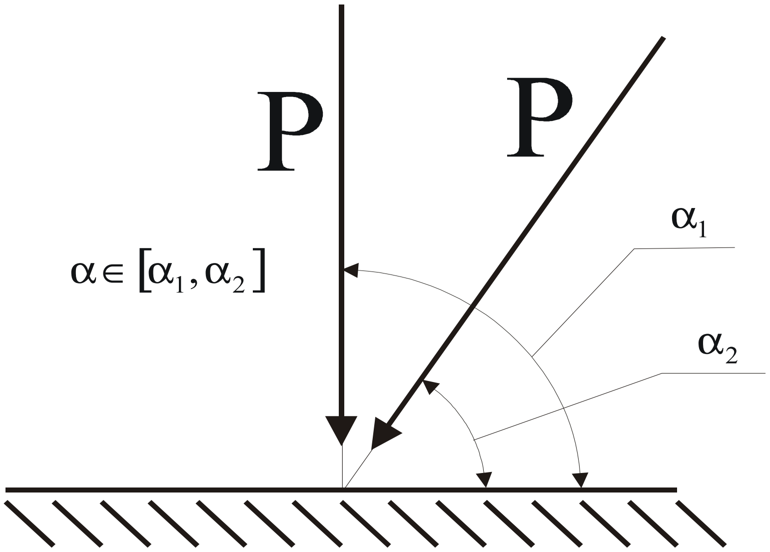
Uncertainties in mechanical systems

Uncertain loads



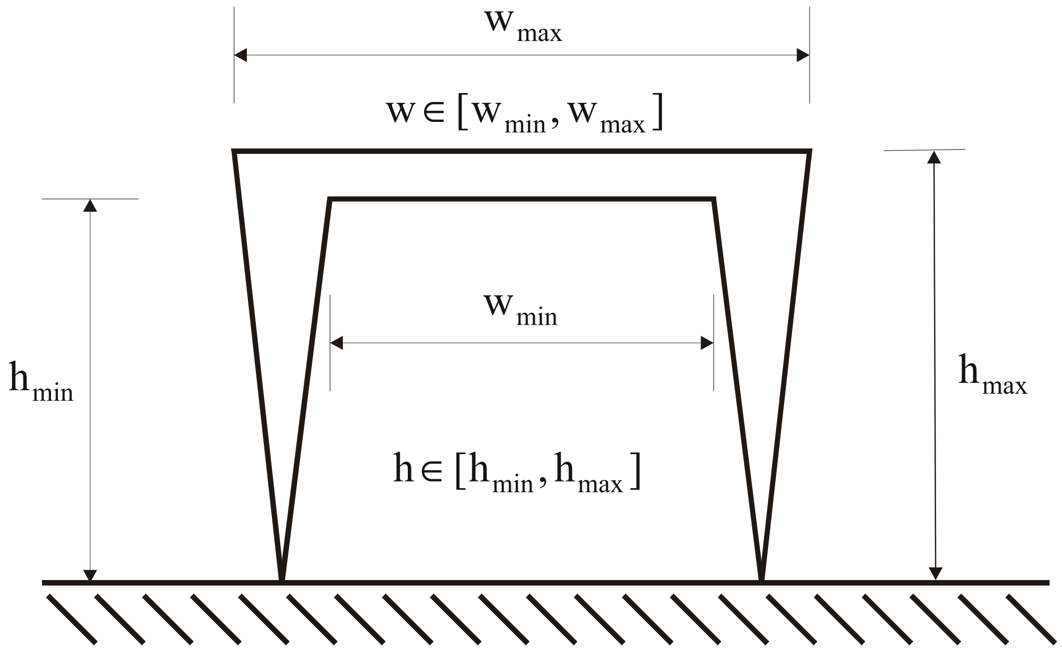
Uncertain material characteristic

$$E \in [E^-, E^+]$$

$$\nu_{xy} \in [\nu_{xy}^-, \nu_{xy}^+]$$

$$G \in [G^-, G^+]$$

Uncertain geometrical parameters



Methods of modelling of uncertain parameters

Deterministic methods

Uncertain parameters are constant $h = h_0$

$g(h_0, \gamma) \geq 0$ γ-safety factor

Probabilistic methods

$$h : \Omega \ni \omega \to h(\omega) \in R$$

Reliability based design of structure

$$R = P\{\omega : g(h(\omega)) \geq 0, \omega \in \Omega\} \geq R_0$$

or

$$P\{g(h) \geq 0\} \geq R_0$$

Worst case analysis

$$h \in [h^-, h^+]$$

$g(h) \geq 0$ for all $h \in [h]$

Random set and fuzzy interval analysis

(Ξ_h, m_h) or $\mu_h : R \to [0,1]$

$$Pl\{\omega : g(\tilde{h}) \geq 0, \tilde{h} \in h(\omega), \omega \in \Omega\} \geq R_0$$

or

$sup\{\mu_g(g) : g \geq 0\} \geq R_0$ $\qquad$ $Pl\{g(h) \geq 0\} \geq R_0$

Modelling of uncertain parameters using random sets

Experiment

In experiment we can measure only upper and lower bounds of some physical quantity h_j

or more general we can measure some convex sets

Results of experiment

$$\Xi = \{A_1, \ldots, A_p\}$$

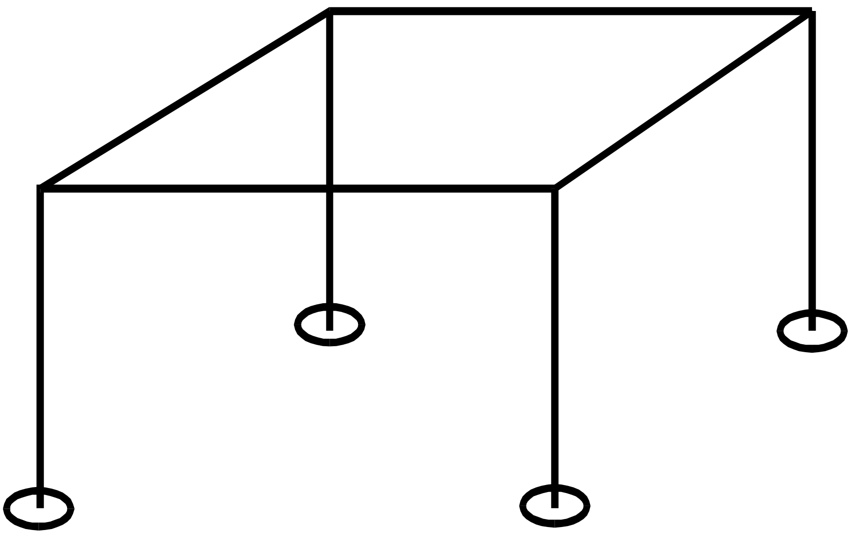
A_i - intervals of convex sets

all results A_i we can obtain with the same probability

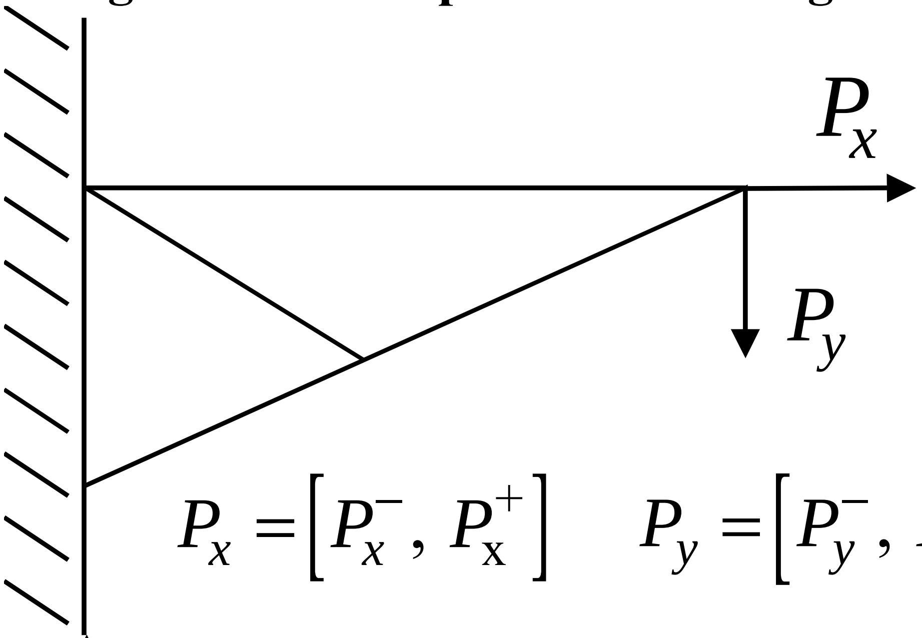
$$m(A_i) = \frac{1}{p}$$

$m(A_i)$ can be viewed as the probability that A_i is the actual range of h_j.

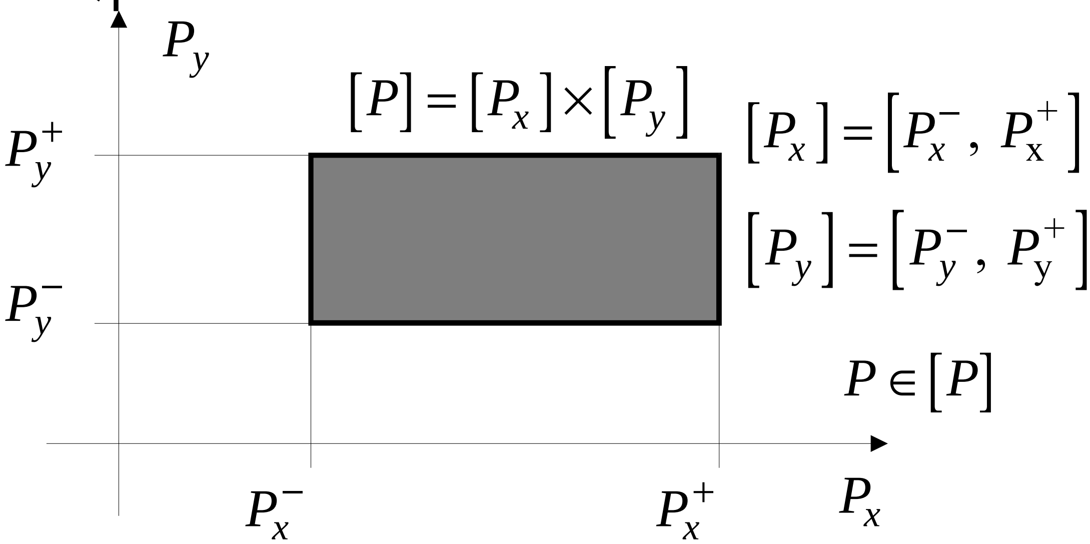
A pair (Ξ, m) is a random set.



Modelling of uncertain parameters using random sets



$$P_x = \left[P_x^-,\ P_x^+\right] \qquad P_y = \left[P_y^-,\ P_y^+\right]$$



$$P_x \in [P_x^-, P_x^+], \quad P_y \in [P_y^-, P_y^+]$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$(\Xi_1, m_1) \qquad\qquad (\Xi_2, m_2)$$

$$\Downarrow$$

$$(\Xi, m)$$

where $\Xi = \Xi_1 \times \Xi_2, \quad m(A_1 \times A_2) = m_1(A_1) \cdot m_2(A_2)$

Random sets

Let us assume that $X = X_1 \times X_2$
and $\forall A \in \Xi,\ A \subset X$

$$A = A_1 \times A_2 \quad \text{where} \quad A_1 \subset X_1,\ A_2 \subset X_2$$

$$A_1 = Proj_1(A) \qquad A_2 = Proj_2(A)$$

$$m_1(A_1) = m_1(Proj_1(A)) = m(A)$$
$$m_2(A_2) = m_2(Proj_2(A)) = m(A)$$

$$(\Xi, m)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$(\Xi_1, m_1) \qquad\qquad (\Xi_2, m_2)$$

Function of random sets

Let $f : X \to Y$
$$f(\Xi_X, m_X) = (\Xi_Y, m_Y)$$

where
$$A_y = f(A_x) \quad A_x \subset X,\ A_y \subset Y$$
$$m_Y(A_y) = m_X(A_x)$$

Upper and lower probability

Given a random set (Ξ,m), a **belief function** *Bel* can be defined as the following set function

$$\forall A \subseteq U, \quad Bel(\mathrm{A}) = \sum \{m(B) : B \subseteq A, \mathrm{B} \in \Xi\}$$

plausibility function defined by

$$\forall \mathrm{A} \subseteq \mathrm{U}, \quad Pl(A) = \sum \{m(B) : B \cap A \neq \emptyset, \mathrm{B} \in \Xi\}$$

It can be shown that

$$Pl(A) = 1 - Bel(\overline{A})$$

When Ξ contains only singletons the *Bel*=*Pl* is a **probability measure** (with finite support).

When Ξ is a nested family i.e.

$$A_1 \subseteq A_2 \subseteq \ldots \subseteq A_p$$

then *Bel* and *Pl* satisfy the **decomposability properties**:

$$Bel(A \cap B) = min\{Bel(A), Bel(B)\}$$
$$Pl(A \cup B) = max\{Pl(A), Pl(B)\}$$

Relation between fuzzy sets and random sets

Fuzzy set F can be defined from any random set (Ξ, m) as follows:

$$\mu_F(u) = \sum_{u \in A} m(A) = Pl(\{u\})$$

When $X = X_1 \times X_2$

$$\mu_F(x_1, x_2) = \sum_{(x_1, x_2) \in A} m(A) = Pl(\{(x_1, x_2)\})$$

Let assume that membership the function μ_F is given. $M(F) = \{\alpha_1, ..., \alpha_p\}$ be the set of membership values such that $\alpha_1 > ... > \alpha_p$. $F(\alpha) = \{u : \mu_F(u) \geq \alpha\}$ be α-level-cut. Then μ_F is equivalent to the unique consonant random set (Ξ, m) defined by:

$$\Xi = \{F(\alpha_i) : i = 1, ..., p\}$$
$$m(F(\alpha_i)) = \alpha_i - \alpha_{i+1}$$

with $\alpha_{p+1} = 0$ by convention.

Extension principle for consonant random sets

Let us assume that $f : X \rightarrow Y$ and (Ξ_X, m_X) are given. Additionally $f(\Xi_X, m_X) = (\Xi_Y, m_Y)$

$$\mu_{F_X}(x) = \sum_{x \in A_x} m(A_x)$$

$$\mu_{F_Y}(y) = \sum_{y \in A_y} m(A_y)$$

It can be shown that for consonant random sets (Ξ_X, m_X) the following relation holds

$$\mu_{F_y}(y) = sup\{\mu_{F_x}(x) : f(x) = y\}$$

When $X = X_1 \times X_2$ for consonant random set we can write

$$\mu_{F_x}(x_1, x_2) = min\{\mu_{F_{x_1}}(x_1), \mu_{F_{x_2}}(x_1)\}$$

and

$$\mu_{F_y}(y) = sup\{min\{\mu_{F_{x_1}}(x_1), \mu_{F_{x_2}}(x_1)\} : f(x_1, x_2) = y\}$$

Extension principle for non-consonant random sets

If random set is non-consonant we can divide family Ξ into parts Ξ^i such that each part Ξ^i is consonant.

For each part Ξ^i fuzzy membership function can be defined

$$\mu^i_{F_x}(x) = \sum_{x \in A_x} m(A_x) \quad \text{where } A_x \in \Xi^i$$

For each part Ξ^i we can apply extension principle

$$\mu^i_{F_y}(y) = sup\{min\{\mu^i_{F_{x_1}}(x_1), \mu^i_{F_{x_2}}(x_1)\} : f(x_1, x_2) = y\}$$

If some members of family Ξ are points, then we can divide this file into the following parts

$\Xi_s^1, \ldots, \Xi_s^k$ - contain sets

Ξ_p - contains points

$$\Xi = \bigcup_i \Xi^i \cup \Xi_p$$

Calculation of upper probability using fuzzy membership function – consonant random set

For consonant random sets (Ξ_x, m_x) upper probability of some event $A_x \subset X$ can be calculated in the following way.

$$Pl(A_x) = sup\{\mu_{F_x}(x) : x \in A_x\}$$

When random set (Ξ_x, m_x) is consonant then random set $(\Xi_y, m_y) = f(\Xi_y, m_y)$ is also consonant and

$$Pl(A_y) = sup\{\mu_{F_y}(y) : y \in A_y\}$$

Using extension principle we can also write

$$Pl(A_y) = sup\{\mu_{F_x}(x) : y = f(x), y \in A_y\}$$

Calculation of upper probability using fuzzy membership function – non-consonant random set

For consonant random sets (Ξ_X, m_X) upper probability of some event $A_X \subset X$ can be calculated in the following way.

$$Pl(A_X) = \sum_i sup\{\mu^i_{F_X}(x) : x \in A_X\}$$

We also can divide family Ξ into the following parts

Ξ_S - contains sets (Ξ_S is consonsnt)

Ξ_P - contains points

$$\Xi = \Xi_S \cup \Xi_P$$

Using the member of the family Ξ_P we can build a histogram $f(x)$.

Using the member of the family Ξ_S we can build a fuzzy membership function

$$\tilde{\mu}(x) = \sum_{x \in A} m(A) \quad \text{where} \quad A \in \Xi_S$$

Calculation of upper probability using fuzzy membership function – non-consonant random set

In this case

$$Pl(A_X) = sup\{\tilde{\mu}(x) : x \in A_X\} + \int_{A_X} f_S(x)dx$$

Let us assume that $X = R$. In this case the members of family Ξ_S are intervals. Using only the left point of these intervals we can built the histogram $f_S(x)$.

Now we can write

$$Pl([a,b]) = \tilde{\mu}(a) + \int_a^b f_S(x)dx + \int_a^b f_P(x)dx$$

If all members of family Ξ are point then

$$\tilde{\mu}(a) \to 0$$

$$\int_a^b f_S(x)dx + \int_a^b f_S(x)dx \to \int_a^b f(x)dx$$

and finally

$$Pl([a,b]) = \int_a^b f(x)dx = P([a,b])$$

Calculation of upper probability using fuzzy membership function – non-consonant random set

If family Ξ_s is non-consonant, then we can divide it into consonant parts Ξ_s^i and then we can write

$$Pl([a,b]) = \sum_i \tilde{\mu}^i(a) + \sum_i \int_a^b f_p^i(x)dx + \int_a^b f_p(x)dx$$

If a set is defined in the space *Y*, then upper probability can be calculated by the use of the extension principle

$$Pl(A_y) = \sum_i sup\{\tilde{\mu}_Y^i(y) : y \in A_y\}$$

$$Pl(A_y) = \sum_i sup\{\tilde{\mu}_x^i(x) : x \in A_x\}$$

where $A_x = f^{-1}(A_y)$.

α-level-cut method

α-level-cut of the fuzzy number F can be defined as the following interval:

$$F_\alpha = \{x : \mu_F(x) \geq \alpha\}$$

If we know α-level-cut of some fuzzy number F then we can calculate fuzzy membership function in the following way:

$$\mu_F(x) = sup\{\alpha : x \in F_\alpha\}$$

This method can be apply, to solution of fuzzy equation. We can use the following algorithm (consonant random set):

1) Calculate α-level-cut of the uncertain parameters $h_{i\alpha}$.
2) Calculate α-level-cut of the solutions $q_{i\alpha}$. In this point interval equations should be solved.
3) Using resolution identity fuzzy membership function of the solution should be calculated
$$\mu_{q_i}(x) = sup\{\alpha : q_i \in q_{i\alpha}\}.$$
4) If we know fuzzy membership functions, then upper and lower probability of the solution can be calculated.

Monotone solutions

General definition of the solution set

$$\mathbf{q}([\mathbf{h}]) = \{\mathbf{q} : \mathbf{K}(\mathbf{h})\mathbf{q} = \mathbf{Q}(\mathbf{h}), \mathbf{h} \in [\mathbf{h}]\}$$

Extreme values of the solution set

$$q_i^- = inf\{q_i : (q_1,...,q_m) \in \mathbf{q}([\mathbf{h}])\}$$
$$q_i^+ = sup\{q_i : (q_1,...,q_m) \in \mathbf{q}([\mathbf{h}])\}$$

Calculation the exact solution of parameter dependent system of equations is NP-hard

If the following functions

$$q_i = q_i(..., h_j, ...)$$

are monotone, then extreme values of the solution set can be calculated using the endpoints of the interval [h].

$$q_i^{\pm} = q_i\left(h_1^{\pm}, ..., h_m^{\pm}\right)$$

Applications of sensitivity analysis

$$\text{If } \frac{\partial q_i(h_1,\dots,h_m)}{\partial h_j} > 0, \quad \text{then}$$

$$q_i^- = q_i(\dots, h_j^-, \dots), \qquad q_i^+ = q_i(\dots, h_j^+, \dots)$$

$$\text{If } \frac{\partial q_i(h_1,\dots,h_m)}{\partial h_j} < 0, \quad \text{then}$$

$$q_i^- = q_i(\dots, h_j^+, \dots), \qquad q_i^+ = q_i(\dots, h_j^-, \dots)$$

We can write it in the following form:

$$q_i^- = q_i\left(\mathbf{h}^{-\operatorname{sign}\left(\frac{\partial \mathbf{q}}{\partial \mathbf{h}}\right)}\right), \qquad q_i^+ = q_i\left(\mathbf{h}^{\operatorname{sign}\left(\frac{\partial \mathbf{q}}{\partial \mathbf{h}}\right)}\right)$$

Interval monotonicity tests

Implicit solutions

It can be shown that if the following interval Jacobian matrices

$$\frac{\partial \hat{\mathbf{F}}([\mathbf{x}],[\mathbf{h}])}{\partial \mathbf{x}} \qquad \frac{\partial \hat{\mathbf{F}}([\mathbf{x}],[\mathbf{h}])}{\partial (x_1,\ldots,h_j,\ldots,x_n)}$$

are regular then solutions of parameter dependent system of equations are monotone.

In order to check regularity of interval matrix interval gauss method Rohn's method could be applied.

Explicit solutions

If

$$0 \notin \frac{\partial \hat{q}_i([\mathbf{h}])}{\partial h_j}$$

then function $q_i = q_i(\ldots,h_j,\ldots)$ is monotone in the interval $[h_j]$.

Implicit function theorem

$$\mathbf{F}(\mathbf{q},\mathbf{h}) = \mathbf{0}$$

$$\sum_{i=1}^{n} \frac{\partial F_k}{\partial q_i} \frac{\partial q_i}{\partial h_j} + \frac{\partial F_k}{\partial h_j} = 0$$

$$\frac{\partial q_i}{\partial h_j} = -\frac{\left| \dfrac{\partial(F_1,\ldots,F_n)}{\partial(q_1,\ldots,h_j,\ldots,q_n)} \right|}{\left| \dfrac{\partial(F_1,\ldots,F_n)}{\partial(q_1,\ldots,q_n)} \right|}$$

$$\frac{\partial \hat{q}_i}{\partial h_j} = -\frac{\left| \dfrac{\partial(\hat{F}_1,\ldots,\hat{F}_n)}{\partial(q_1,\ldots,h_j,\ldots,q_n)} \right|}{\left| \dfrac{\partial(\hat{F}_1,\ldots,\hat{F}_n)}{\partial(q_1,\ldots,q_n)} \right|}$$

An interval extension of the Jacobian matrices

$$\left\{ \left| \frac{\partial(F_1,\ldots,F_n)}{\partial(q_1,\ldots,q_n)}(\mathbf{q}) \right| : \mathbf{q} \in [\mathbf{q}] \right\} \subseteq$$

$$\subseteq \left| \frac{\partial\left(\hat{F}_1,\ldots,\hat{F}_n\right)}{\partial(q_1,\ldots,q_n)}([\mathbf{q}]) \right|$$

$$\left\{ \left| \frac{\partial(F_1,\ldots,F_n)}{\partial\left(q_1,\ldots,h_j,\ldots,q_n\right)}(\mathbf{q}) \right| : \mathbf{q} \in [\mathbf{q}] \right\} \subseteq$$

$$\subseteq \left| \frac{\partial\left(\hat{F}_1,\ldots,\hat{F}_n\right)}{\partial\left(q_1,\ldots,h_j,\ldots,q_n\right)}([\mathbf{q}]) \right|$$

Algorithm of calculation

$$\frac{\partial(F_1,...,F_n)}{\partial(q_1,...,q_n)}([\mathbf{q}]) \text{ are regular}$$

$$\frac{\partial(F_1,...,F_n)}{\partial(q_1,...,h_j,...,q_n)}([\mathbf{q}]) \text{ are regular}$$

$$\Downarrow$$

$$sgin\left(\frac{\partial\hat{q}_j([\mathbf{h}])}{\partial h_j}\right) = const$$

$$\Downarrow$$

$$q_i(h_j) \text{ are monotone}$$

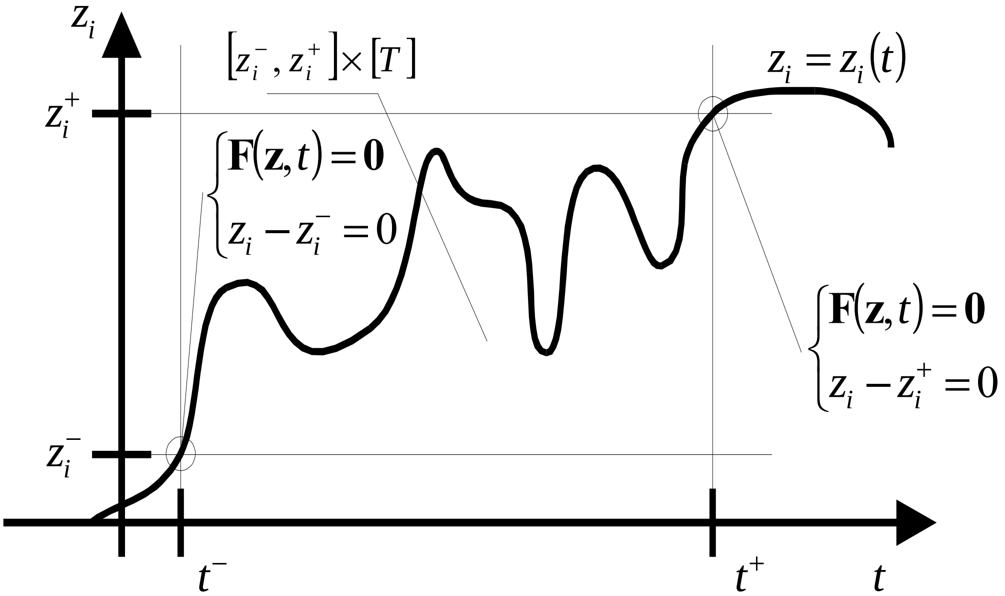
$$\Downarrow$$

$$q_i^{\pm} = q_i(h_1^{\pm},...,h_m^{\pm})$$

If an equation

$$\mathbf{g}(\mathbf{x}) = \mathbf{0}$$

has more than one solutions in the interval [x], then a matrix $\hat{\mathbf{g}}([\mathbf{x}])$ can not be regular.



$$\begin{cases} \mathbf{F}(\mathbf{z}, \mathrm{t}) = \mathbf{0} \\ \mathrm{z_i} - \mathrm{z_i^{\mp}} = 0 \end{cases} \Rightarrow \begin{cases} \mathbf{z} = \mathbf{z}(\mathrm{t}) \\ \mathrm{z_i} - \mathrm{z_i^{\pm}} = 0 \end{cases}$$

$$\mathbf{F}\left(\mathrm{z_1}, \ldots, \mathrm{z_i^{\pm}}, \ldots, \mathrm{z_n}, \mathrm{t}\right) = \mathbf{0}$$

1)

$$z_i^- = \min\{z_i(t^-), z_i(t^+)\}$$

$$z_i^+ = \max\{z_i(t^-), z_i(t^+)\}$$

2)

$$\frac{\partial \hat{F}([z_1], \ldots, z_i^{\pm}, \ldots, [z_n], [t])}{\partial(z_1, \ldots, z_{i-1}, t, z_{i+1}, \ldots, z_n)};$$

3) exist $t^* \in [t]$, such that $(z(t^*), t^*) \in \mathrm{int}([\mathbf{x}])$;

then

$$z_i^- = \inf\{z_i(t) : t \in [t]\}$$

$$z_i^+ = \sup\{z_i(t) : t \in [t]\}$$

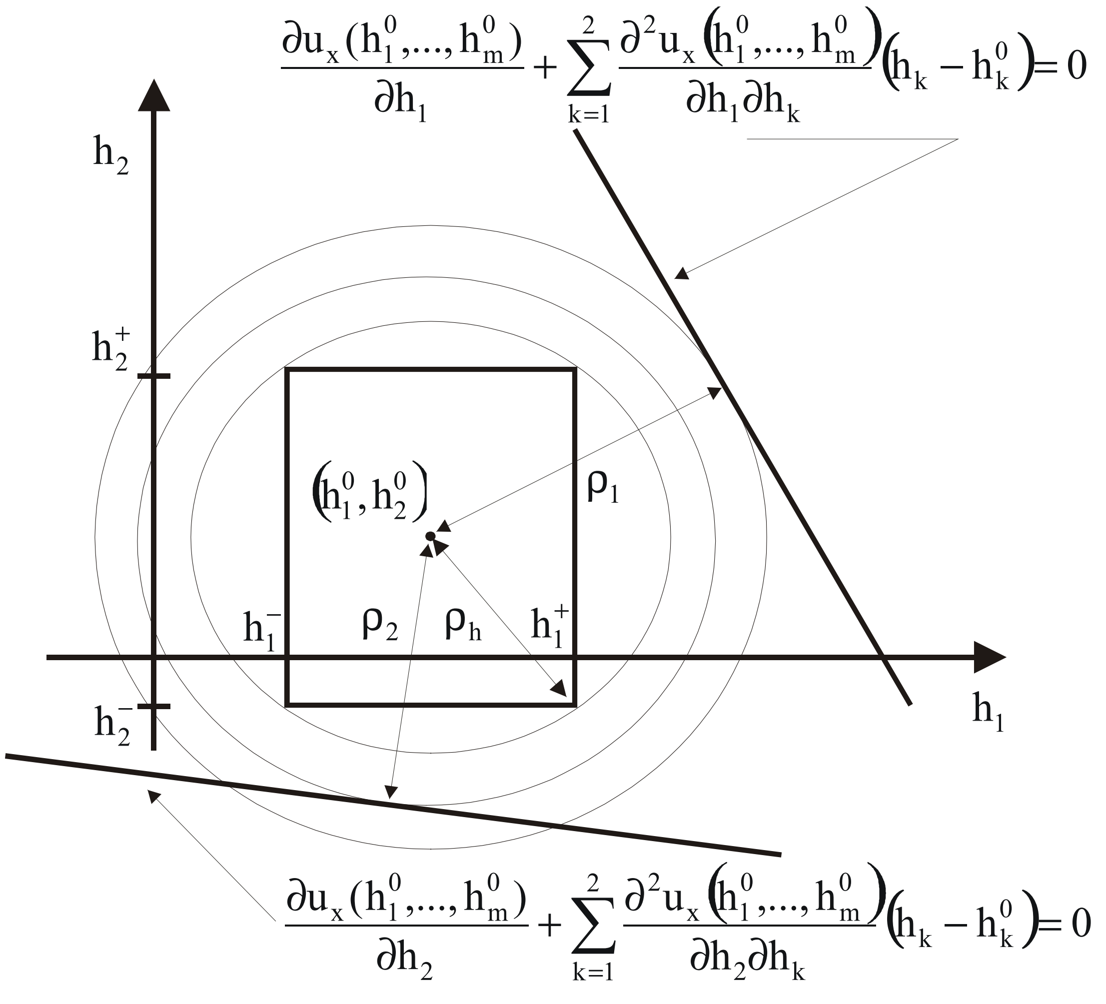
Point monotonicity tests

First method

$$\frac{\partial u_x(h_1,\dots,h_m)}{\partial h_j} \approx \frac{\partial u_x(h_1^0,\dots,h_m^0)}{\partial h_j} + \sum_{k=1}^{m} \frac{\partial^2 u_x\left(h_1^0,\dots,h_m^0\right)}{\partial h_j \partial h_k}\left(h_k - h_k^0\right) = 0$$

A distance of this hypersurface from the point $\left(h_1^0,\dots,h_m^0\right)$ is as follows.

$$\rho_j = \left|\frac{\partial u_x(h_1^0,\dots,h_m^0)}{\partial h_j}\right| \Bigg/ \sqrt{\sum_{k=1}^{m}\left(\frac{\partial^2 u_x\left(h_1^0,\dots,h_m^0\right)}{\partial h_j \partial h_k}\right)^2}$$



Point monotonicity test

Second method

$$\frac{\partial \breve{u}_x(h_1,\ldots,h_m)}{\partial h_j} = \frac{\partial u_x(h_1^0,\ldots,h_m^0)}{\partial h_j} + \sum_{k=1}^{m} \frac{\partial^2 u_x(h_1^0,\ldots,h_m^0)}{\partial h_j \partial h_k}(h_k - h_k^0)$$

If

$$\mathrm{sign}\left(\frac{\partial \breve{u}_x(h_1^{\pm},\ldots,h_m^{\pm})}{\partial h_j}\right) = \mathrm{const}$$

then

$$\left\{(h_1,\ldots,h_m): \frac{\partial \breve{u}_x(h_1,\ldots,h_m)}{\partial h_j} = 0\right\} \cap [\mathbf{h}] = \varnothing$$

i.e. $\mathrm{sign}\left(\frac{\partial \breve{u}_x(h_1,\ldots,h_m)}{\partial h_j}\right) = \mathrm{const}$ for $h_i \in [h_i]$ i=1,…,m

and function $u_x(h_1^0,\ldots,h_{j-1}^0,h_j,h_{j+1}^0,\ldots,h_m^0)$ is monotone in the interval

$[h_j]$ where $h_i^0 \in [h_i]$ i=1,…,m and $i \neq j$.

Point monotonicity test

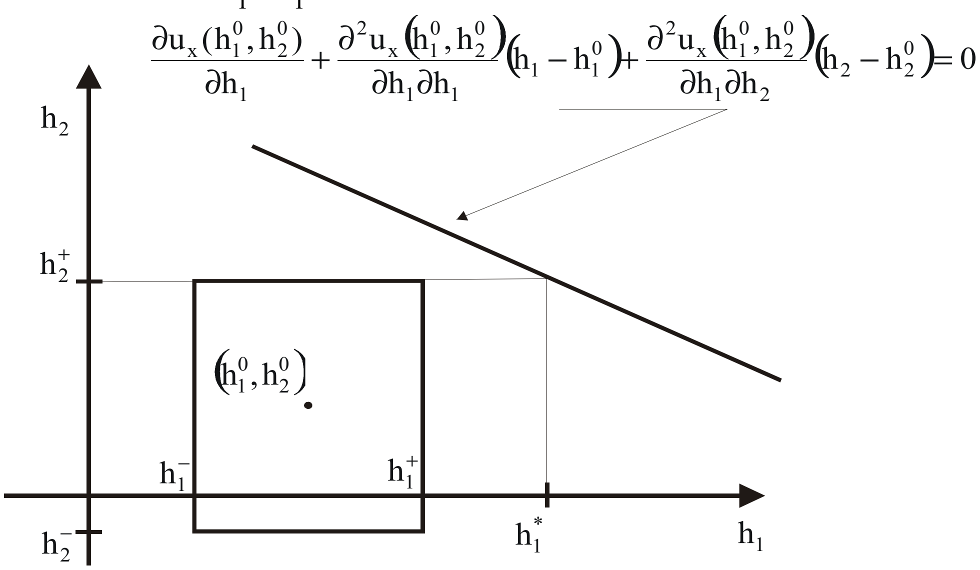
Third method

Let m=2 and $h_2 = h_2^+$

$$\frac{\partial u_x(h_1^0, h_2^0)}{\partial h_1} + \frac{\partial^2 u_x\left(h_1^0, h_2^0\right)}{\partial h_1 \partial h_1}\left(h_1 - h_1^0\right) + \frac{\partial^2 u_x\left(h_1^0, h_2^0\right)}{\partial h_1 \partial h_2}\left(h_2^+ - h_2^0\right) = 0$$

then

$$h_1^* = h_1^0 - \frac{1}{\dfrac{\partial^2 u_x\left(h_1^0, h_2^0\right)}{\partial h_1 \partial h_1}}\left(\frac{\partial u_x(h_1^0, h_2^0)}{\partial h_1} + \frac{\partial^2 u_x\left(h_1^0, h_2^0\right)}{\partial h_1 \partial h_2}\left(h_2^+ - h_2^0\right)\right)$$



In multidimensional case:

$$h_i^* = h_i^0 - \frac{1}{\dfrac{\partial^2 u_x\left(h_1^0, \ldots, h_m^0\right)}{\partial h_j \partial h_i}}\left(\frac{\partial u_x(h_1^0, h_2^0)}{\partial h_j} + \sum_{\substack{k=1 \\ k \neq i}}^{m} \frac{\partial^2 u_x\left(h_1^0, \ldots, h_m^0\right)}{\partial h_j \partial h_k}\left(h_k^{\pm} - h_k^0\right)\right)$$

If

$$h_i^* \notin [h_i]$$

then function $u_x\left(h_1^0, \ldots, h_{j-1}^0, h_j, h_{j+1}^0, \ldots, h_m^0\right)$ is monotone in the interval $[h_j]$ where $h_i^0 \in [h_i]$ i=1,…,m and $i \neq j$.

Point monotonicity test

Forth method

$$\frac{\partial u_x\left(h_j\right)}{\partial h_j} := \frac{\partial}{\partial h_j} u_x\left(h_1^0, \ldots, h_{j-1}^0, h_j, h_{j-1}^0, \ldots, h_m^0\right)$$

$$\frac{\partial u_x(h_j)}{\partial h_j} \approx \frac{\partial u_x(h_j^0)}{\partial h_j} + \frac{\partial^2 u_x\left(h_j^0\right)}{\partial h_j^2}\left(h_j - h_j^0\right) = 0$$

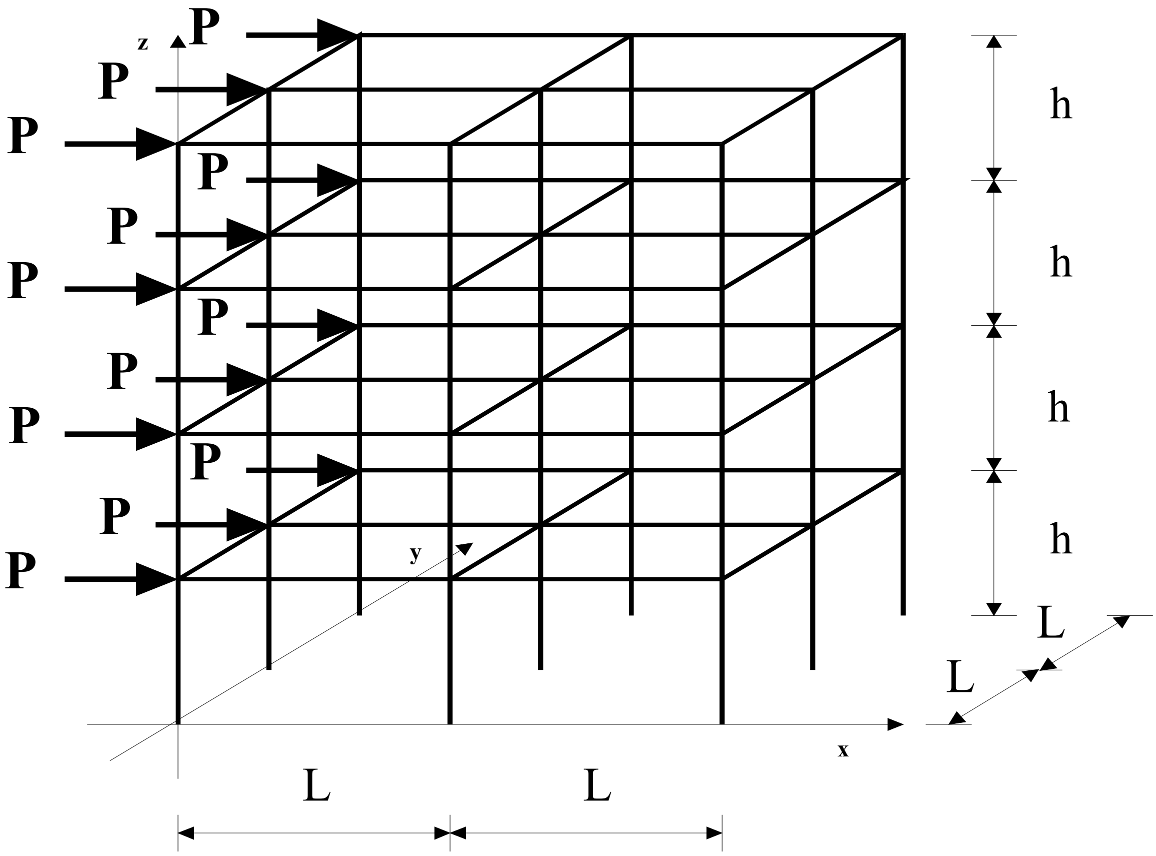
$$h_j = h_j^0 - \frac{\dfrac{\partial u_x(h_j^0)}{\partial h_j}}{\dfrac{\partial^2 u_x\left(h_j^0\right)}{\partial h_j^2}}$$

$$\frac{\partial u_x(h_j^0)}{\partial h_j} \approx \frac{u_x(h_j^0 + \Delta h_j^0) - u_x(h_j^0)}{\Delta h_j^0}$$

$$\frac{\partial^2 u_x(h_j^0)}{\partial h_j^2} \approx \frac{u_x(h_j^0 + \Delta h_j^0) - 2 \cdot u_x(h_j^0) + u_x(h_j^0 - \Delta h_j^0)}{\left(\Delta h_j^0\right)^2}$$

$$h_j^* \approx h_j^0 - \frac{\left[u_x(h_j^0 + \Delta h_j^0) - u_x(h_j^0)\right] \cdot \Delta h_j^0}{u_x(h_j^0 + \Delta h_j^0) - 2 \cdot u_x(h_j^0) + u_x(h_j^0 - \Delta h_j^0)}$$

$$h_j^* \notin \left[h_j^-, h_j^+\right] = \left[h_j\right]$$



$A = 0.05^2\ [m^2],\ L = h = 3\ [m]$, **P**=[1, 2] [kN].

Numerical results

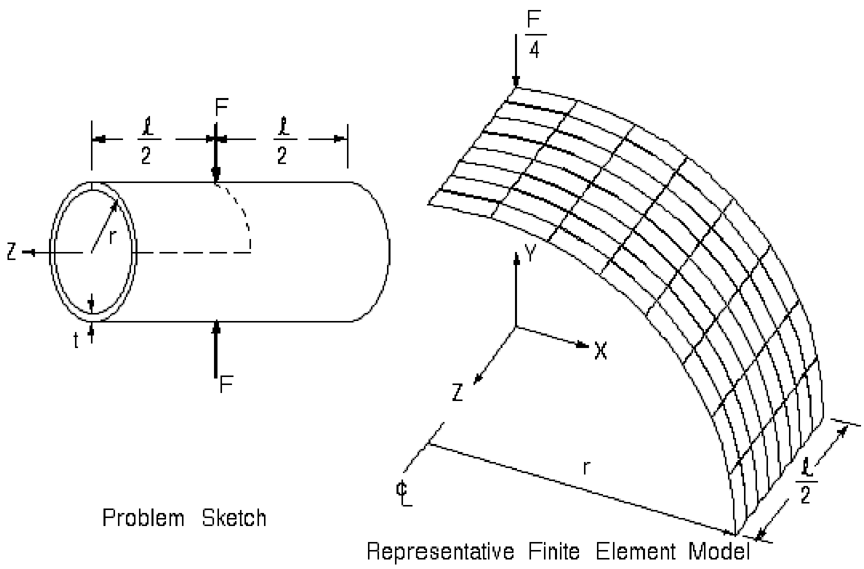
ne	42	63	160	260	325	390
dim	108	162	384	600	750	900
q^{-}_{max}	0.0396	0.0886	0.1104	0.0854	0.1321	0.1890
q^{+}_{max}	0.0792	0.1773	0.2209	0.1708	0.2643	0.3780

Thin-walled cylinder

Data

$E \in [2.0 \cdot 10^5, 2.2 \cdot 10^5]$ MPa, $\nu \in [0.2, 0.3]$

L=0.263 m, r=0.126 m, t=$2.38 \cdot 10^{-3}$ m, F=444.8 N



$$\Delta u_y \overset{df}{=} u_y(h + \Delta h) - u_y(h)$$

$$\Delta^2 u_y \overset{df}{=} u_y(h + \Delta h) - 2 \cdot u_y(h) + u_y(h - \Delta h)$$

$$\Delta h^* \overset{df}{=} \frac{[u_y(h + \Delta h) - u_y(h)] \cdot \Delta h}{u_y(h + \Delta h) - 2 \cdot u_y(h) + u_y(h - \Delta h)}$$

Because

$$2 \cdot 10^{10} \text{ MPa} = E^+ - E^- << \left|\Delta h_E^*\right| = 9.972 \cdot 10^4 \text{ MPa}$$

$$0.1 = \nu^+ - \nu^- << \left|\Delta h_\nu^*\right| = 2.662$$

then functions $u_y(E), u_y(\nu)$ are monotone in the intervals [E],[ν].

The interval solution $u_y \in [-0.043514, -0.03748]$.

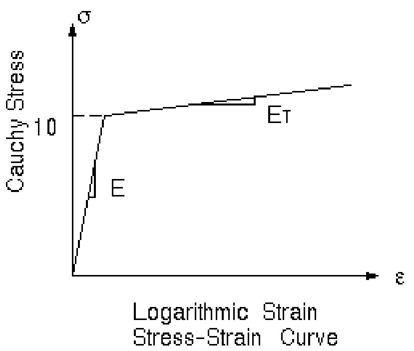
Large strain in-plane torsion

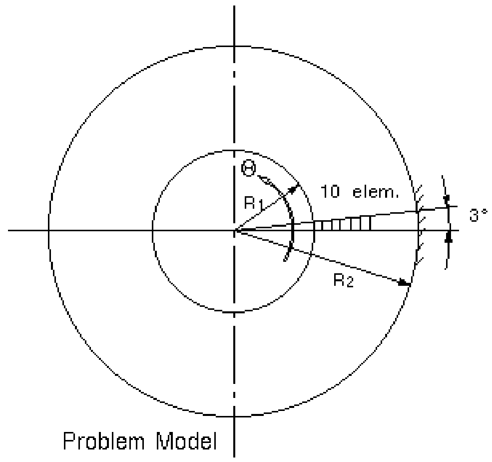
Data

$E \in [49.3, 49.9]$ MPa $\quad E_T \in 0.28$ MPa $\quad \sigma_y = 0.6$ MPa

$\nu \in [0.32, 0.34]$ $\quad R_1 = 0.254$ m $\quad R_2 = 0.508$ m

$\Theta = 60^o$





We assume that

$$h_1 = E, \quad h_2 = \nu$$

Because

$$h_i^* \notin \left[h_i^-, h_i^+\right] \quad \text{for} \quad i = 1,2$$

then the functions

$$\sigma_x = \sigma_x(h_1), \quad \sigma_x = \sigma_x(h_2)$$

are monotone in the intervals $[h_1], [h_2]$.

Interval solution

$$\sigma_x \in [0.36, 0.48] \text{MPa}$$

HEAT TRANSFER

$$\hat{\mathbf{K}}([\boldsymbol{\lambda}])\mathbf{q} = \hat{\mathbf{Q}}([\boldsymbol{\lambda}])$$
$$\hat{\mathbf{q}}([\mathbf{h}]) = \text{hull}\,\Sigma\left(\hat{\mathbf{K}}([\mathbf{h}]), \hat{\mathbf{Q}}([\mathbf{h}])\right)$$

$$\hat{\mathbf{K}}([\boldsymbol{\lambda}])\frac{\partial \mathbf{q}}{\partial \mathbf{h}} = \frac{\partial \hat{\mathbf{Q}}([\boldsymbol{\lambda}])}{\partial \mathbf{h}} - \frac{\partial \hat{\mathbf{K}}([\boldsymbol{\lambda}])}{\partial \mathbf{h}}\hat{\mathbf{q}}([\mathbf{h}])$$

$$\frac{\partial \hat{\mathbf{q}}([\mathbf{h}])}{\partial \mathbf{h}} = \text{hull}\,\Sigma\left(\hat{\mathbf{K}}([\mathbf{h}]), \frac{\partial \hat{\mathbf{Q}}([\mathbf{h}])}{\partial \mathbf{h}} - \frac{\partial \hat{\mathbf{K}}([\mathbf{h}])}{\partial \mathbf{h}}\hat{\mathbf{q}}([\mathbf{h}])\right)$$

$$[\mathbf{h}] = \bigcup_i [\mathbf{h}_i], \qquad \frac{\partial \hat{\mathbf{q}}([\mathbf{h}])}{\partial \mathbf{h}} \subseteq \bigcup_i \frac{\partial \hat{\mathbf{q}}([\mathbf{h}_i])}{\partial \mathbf{h}}$$

$$\begin{cases} R_1 < r < R_2 : \frac{1}{r}\frac{d}{dr}\left(r\lambda\frac{dT(r)}{dr}\right) + Q = 0 \\ r = R_1 \quad : -\lambda\frac{dT(r)}{dr} = \alpha\left(T(r) - T_b\right) \\ r = R_2 \quad : T(r) = T_t \end{cases}$$

R_1**=0.0005 [m],** R_2**=10***R_1**,** α**=2000,** T_b**= 32 [**$^\circ$C**],** T_t**=37[**$^\circ$C**],** ***Q*****=10245[W/**m^3**]m,** $\lambda \in$ **[0.21,0.23][W/mK].**

	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
$\underline{T}_i$ [$^\circ$C]	36.586	35.470	34.782	34.284	33.894	33.573	33.302	33.065	32.857	32.669	32.500
$\overline{T}_i$ [$^\circ$C]	36.619	35.494	34.800	34.298	33.905	33.582	33.308	33.070	32.859	32.671	32.500

Conclusions

1) Presented methods are very effective tools for modelling of structures with uncertain parameters.

2) Presented method can be applied when intervals $[h_j]$ are sufficiently narrow.

3) Presented algorithms can be extend to the case when uncertain parameters will be fuzzy numbers.

4) These methods can be applied to nonlinear problems of computational mechanics.

5) Presented algorithms use results generated by existing engineering software.

6) Presented methods are universally applicable.

7) When uncertain parameters are fuzzy numbers presented method can be applied to calculation upper and lower probability of the safety of structures.

8) When intervals are too wide then the interval monotonicity test or the Monte-Carlo simulations should be applied. Unfortunately these methods have a higher computational complexity.