XXXII SYMPOSIUM
ON MATHEMATICAL PHYSICS

Calculation of reliability of structures using fuzzy sets theory

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Relations between random and fuzzy sets

$$(\Xi, m) \Rightarrow \mu_F$$

Fuzzy set F can be defined from any random set (Ξ, m) as follows:

$$\mu_F(u) = \sum_{u \in A} m(A) = Pl(\{u\}) \quad (1)$$

$$\mu_F \Rightarrow (\Xi, m) \quad (2)$$

Let assume that membership the function μ_F is given.
$M(F) = \{\alpha_1, \ldots, \alpha_p\}$ be the set of membership values such that $\alpha_1 > \ldots > \alpha_p$. $F(\alpha) = \{u : \mu_F(u) \geq \alpha\}$ be α-level-cut.
Then μ_F is equivalent to the unique consonant random set (Ξ, m) defined by:

$$\Xi = \{F(\alpha_i) : i = 1, \ldots, p\}, \quad m(F(\alpha_i)) = \alpha_i - \alpha_{i+1} \quad (3)$$

with $\alpha_{p+1} = 0$ by convention.

Upper and lower probability

It can be shown that:

$$Bel(A) \leq P(A) \leq Pl(A) \quad \text{for all } A \in \Sigma \quad (4)$$

This equation provides for the definition of two upper and lower distribution function:

$$F_*(x) = Bel([-\infty, x]), \quad F^*(x) = Pl([-\infty, x]) \quad (5)$$

Estimation of upper probability of failure based on the fuzzy arithmetic

$$P_f^+ = 1 - R^- = sup\{\mu(\mathbf{x}) : g(\mathbf{x}) < 0\} \quad (6)$$

Evaluation of fuzzy constants form uncertain experimental data

Let mechanical system is described by the following equation:

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{a}) \quad (7)$$

where $\mathbf{f} : R^p \times R^m \to R^q$ and $\mathbf{a} \in R^m$ is a vector of unknown constants.

$$a_{i\alpha}^- = inf\{a_i(\vec{x}, \vec{y}) : \vec{x} \in \vec{\mathbf{x}}_\alpha, \vec{y} \in \vec{\mathbf{y}}_\alpha\} \quad (8)$$

$$a_{i\alpha}^+ = sup\{a_i(\vec{x}, \vec{y}) : \vec{x} \in \vec{\mathbf{x}}_\alpha, \vec{y} \in \vec{\mathbf{y}}_\alpha\} \quad (9)$$

Using these α-level-cuts we can define fuzzy numbers A_i:

$$\mu_{A_i}(a_i) = sup\{\alpha : a_\mathrm{i} \in [a_{i\alpha}^-, a_{i\alpha}^+]\} \quad (10)$$

Fuzzy experimental data

$$\left\lfloor \sigma_{ij}^{ex} \right\rfloor = \left\lfloor \sigma_{ij}^{-}, \sigma_{ij}^{+} \right\rfloor \quad i = 1,\ldots,n, \quad j = 1,\ldots,m_i \tag{11}$$

n – number of stress which was measured
m_i - number of repetition in measurement of the stress σ_i

$$\mu(\sigma_i) = \sum_{\sigma_i \in [\sigma_{ij}^{-}, \sigma_{ij}^{+}]} p_{ij} \qquad j=1,\ldots,m_i \tag{12}$$

$$p_{ij} = m_i\left(\left[\sigma_{ij}\right]\right) = \frac{1}{m_i} \quad \left(\sum_{j=1}^{m_i} p_{ij} = 1 \quad \text{for } i=1,\ldots,n\right) \tag{13}$$

(Ξ_i, m_i) - random sets which contain experimental data
We assume that

$$\bigcap_{j=1}^{m_i} \left[\sigma_{ij}^{ex}\right] = \emptyset \qquad \mathrm{i}=1,\ldots,n \tag{14}$$

Using fuzzy set (12) we can obtain equivalent random sets $(\hat{\Xi}_i, \hat{m}_i)$ which is consonant. It can be shown that:

$$Pl(A) = \sum\{m(B) : B \cap A \neq \emptyset, \mathrm{B} \in \Xi\} = \sum\{\hat{m}(B) : B \cap A \neq \emptyset, \mathrm{B} \in \hat{\Xi}\} \tag{15}$$

and

$$\mu_F(u) = \sum_{\substack{u \in A \\ A \in \Xi}} m(A) = \sum_{\substack{u \in A \\ A \in \hat{\Xi}}} \hat{m}(A) = Pl(\{u\}) \tag{16}$$

$$P_f^{\,+} = \sum\{m(A_i) : (-\infty, 0) \cap g(A_i) \neq \emptyset, A_i \in \Xi\} = \tag{17}$$

$$= \sum\{m(A_i) : (-\infty, 0) \cap g(A_i) \neq \emptyset, A_i \in \hat{\Xi}\} \tag{18}$$

Conclusions

In this paper a new method of modelling of uncertain parameters was presented. This method can be used instead of the Monte-Carlo simulation (based on uncertain data).

This method is based on relation between the theory of random sets and the theory of fuzzy sets.

This method is based on the α-level-cuts of fuzzy numbers because of this it can be used in the fuzzy finite element method and fuzzy boundary element method.

If monotonicity tests can be applied successfully, then this method can be used in modelling of nonlinear problems of computational mechanics.

A new method for calculation of fuzzy material constant is presented. This method is based on uncertain experimental data (intervals or convex sets) and the last squares method. In calculation α-level-cuts and monotonicity test were applied. The method can be extended on the case of fuzzy experimental data.