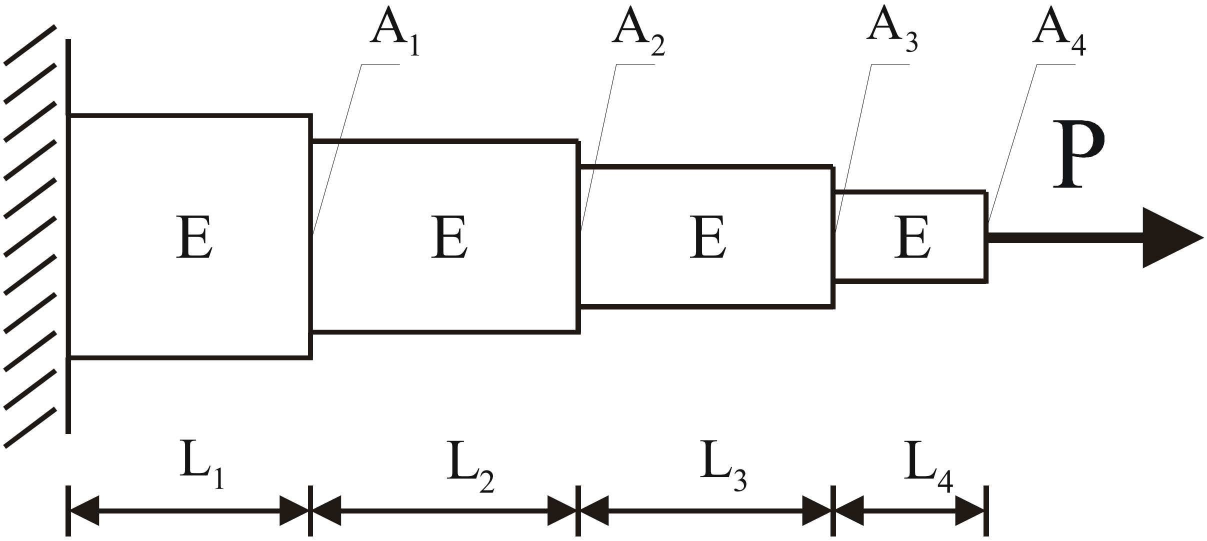
XXXIX SYMPOZJON
„MODELOWANIE W MECHANICE"

ANDRZEJ POWNUK

APPLICATIONS OF REGULAR INTERVAL JACOBIAN MATRICES TO MODELLING OF MECHANICAL SYSTEMS WITH UNCERTAIN PARAMETERS

POLITECHNIKA ŚLĄSKA
WYDZIAŁ BUDOWNICTWA
KATEDRA MECHANIKI TEORETYCZNEJ

MODELOWANIE NIEPEWNOŚCI PARAMETRÓW W UKŁADACH MECHANICZNYCH



Przedziałowe parametry:

$$E \in \left[E^-, E^+\right] \mathrm{MPa}$$

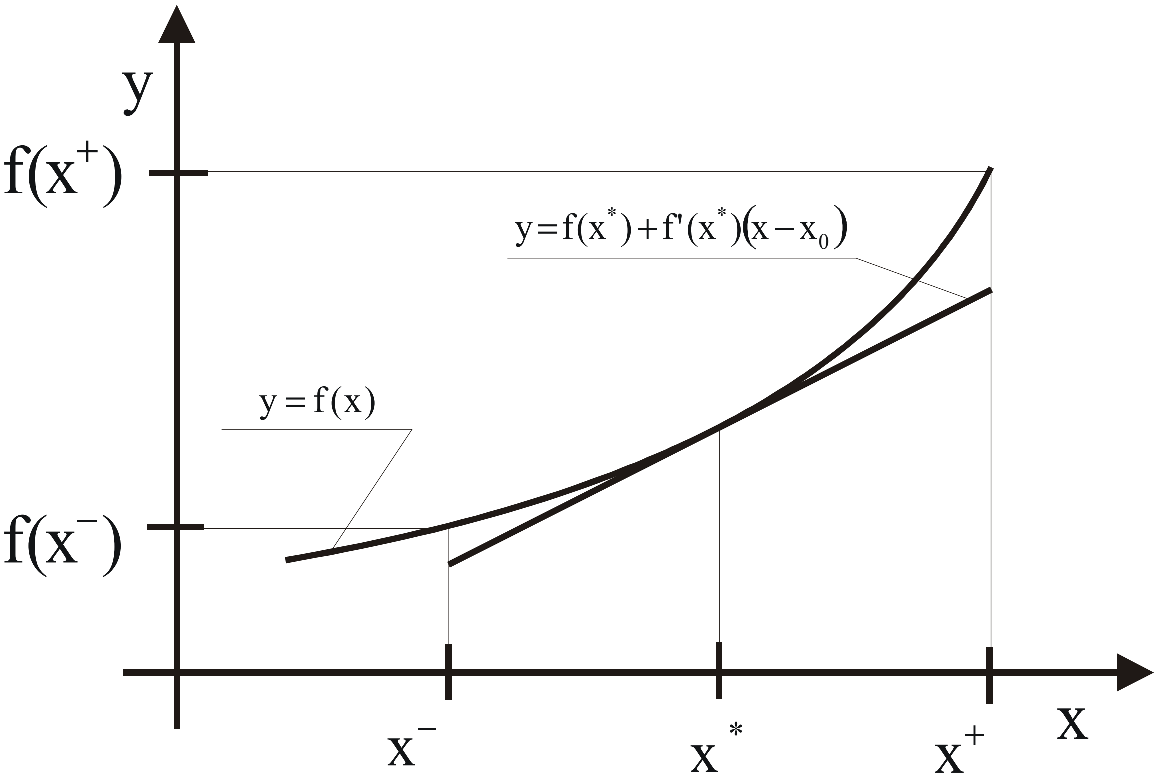
$$A_i \in \left[A_i^-, A_i^+\right] \mathrm{m}^2$$

$$P \in \left[P^-, P^+\right] \mathrm{kN}$$

$$\mathbf{q}([\mathbf{h}]) = \{\mathbf{q} : \mathbf{K}(\mathbf{h})\mathbf{q} = \mathbf{Q}(\mathbf{h}), \mathbf{h} \in [\mathbf{h}]\}$$

$$[\mathbf{q}] = \mathrm{hull}\ \mathbf{q}([\mathbf{h}])$$

BADANIE MONOTONICZNOŚCI



$$f(x) \approx f(x^{*}) + f'(x^{*})(x - x^{*})$$

$$f'(x) \approx f'(x^{*})$$

Jeśli $|f'(x^{*})| >> 0$,
to f(x) jest **monotoniczna**
w otoczeniu punktu x^{*}

STATYKA UKŁADÓW LINIOWO-SPRĘŻYSTYCH

$$\mathbf{K}(\mathbf{h})\mathbf{q}(\mathbf{h}) = \mathbf{Q}(\mathbf{h})$$

$$\mathbf{K}(\mathbf{h})\frac{\partial \mathbf{q}(\mathbf{h})}{\partial \mathbf{h}} = \frac{\partial \mathbf{Q}(\mathbf{h})}{\partial \mathbf{h}} - \frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}}\mathbf{q}(\mathbf{h})$$

Jeśli $\dfrac{\partial q_i(\mathbf{h})}{\partial h_j} > 0$, to $q_i^+ = q_i\left(\ldots, h_j^+, \ldots\right)$

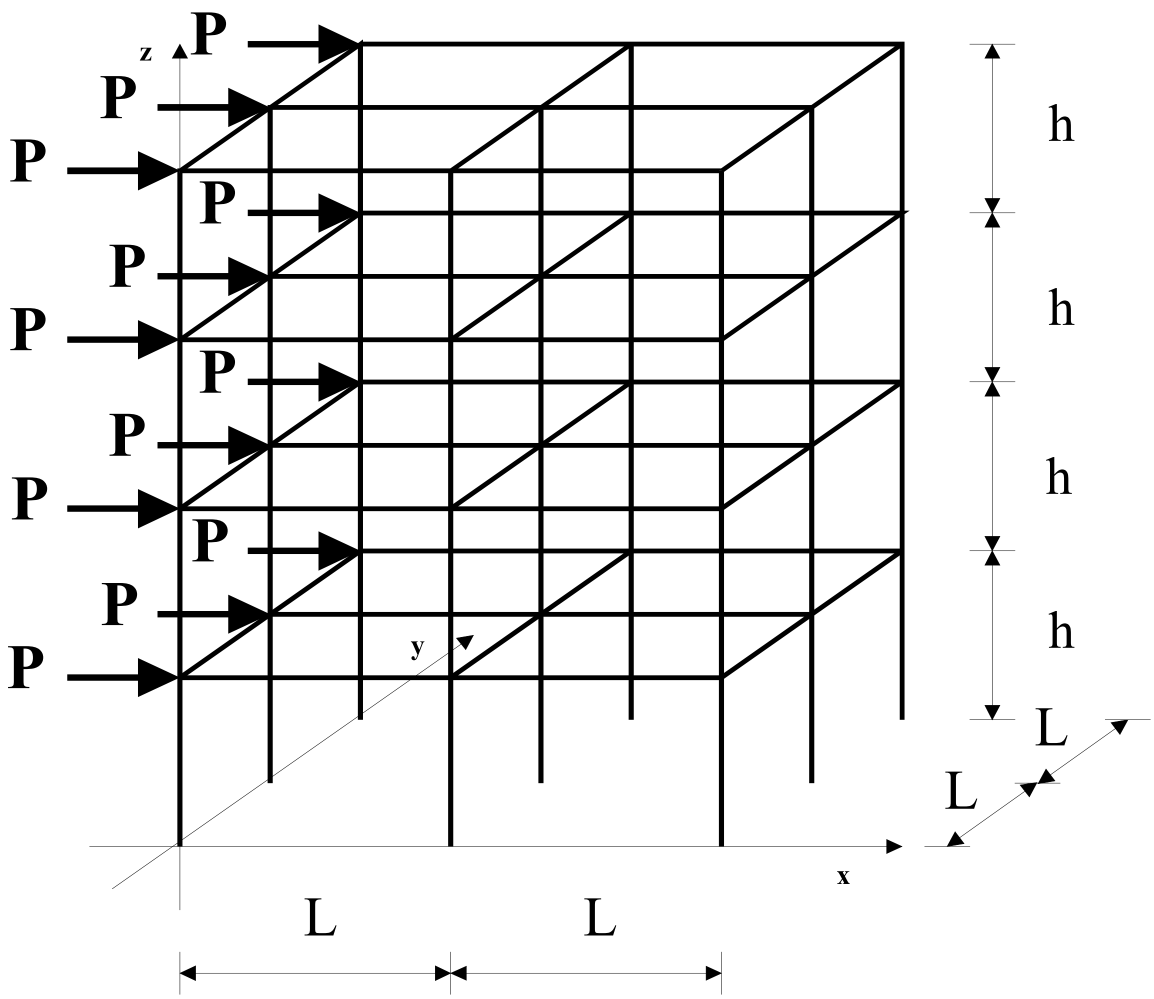
Jeśli $\dfrac{\partial q_i(\mathbf{h})}{\partial h_j} < 0$, to $q_i^- = q_i\left(\ldots, h_j^-, \ldots\right)$

W skrócie

$$\mathbf{q}^- = \mathbf{q}\left(\mathbf{h}^{-\operatorname{sgn}\left(\frac{\partial \mathbf{q}}{\partial \mathbf{h}}\right)}\right), \qquad \mathbf{q}^+ = \mathbf{q}\left(\mathbf{h}^{\operatorname{sgn}\left(\frac{\partial \mathbf{q}}{\partial \mathbf{h}}\right)}\right)$$

ANALOGICZNE ZWIĄZKI ZACHODZĄ W PRZYPADKU ZAGADNIEŃ DYNAMICZNYCH ORAZ NIELINIOWYCH.

MODELOWANIE UKŁADÓW PRĘTOWYCH Z PRZEDZIAŁOWYMI PARAMETRAMI



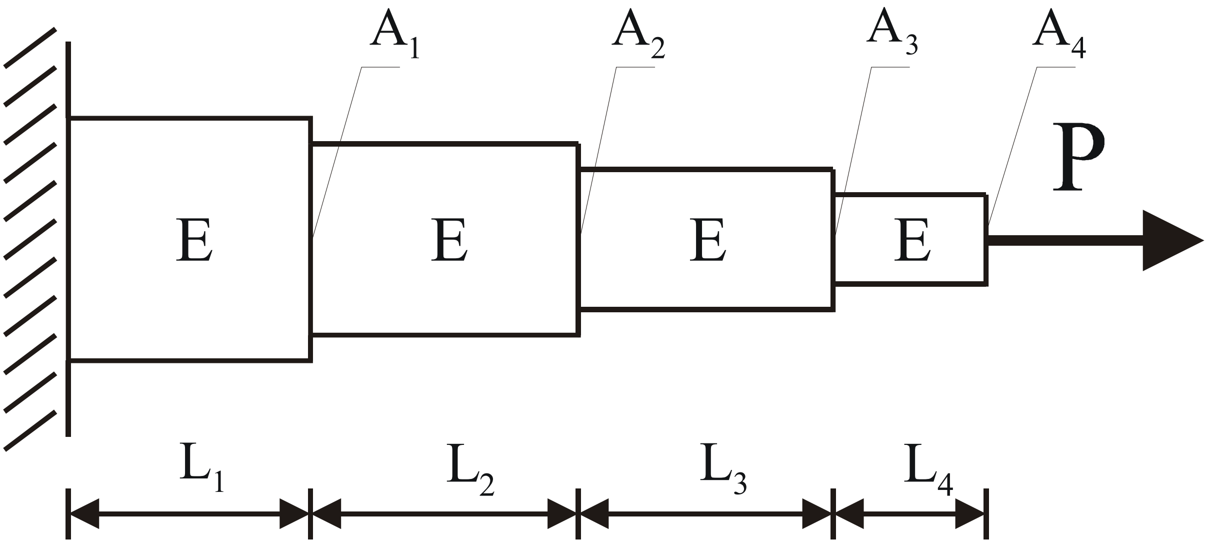
$E = 2.0 \cdot 10^5$ [MPa], $G = 8.5 \cdot 10^{10}$ [MPa], $J_y = J_z = \frac{0.05^4}{12}$ [m^4],

$J_0 = 0.14 \cdot 0.05^4$ [m^4], $A = 0.05^2$ [m^2], L=h=3 [m],

P=[1, 2] [kN]

Liczba stopni swobody układu: 900 (390 prętów)
Liczba przedziałowych parametrów: 30
Maksymalne przemieszczenie: q_{max} =[0.30, 0.37] [m]

ROZCIĄGANIE PRĘTÓW WYKONANYCH Z MATERIAŁU SPRĘŻYSTO-PLASTYCZNEGO ZE WZMOCNIENIEM KINEMATYCZNYM I IZOTROPOWYM



W obliczeniach przyjęto następujące dane liczbowe:

$L_1 = L_2 = L_3 = L_4 = 1$ m, $E \in \left[2.0 \cdot 10^5, 2.2 \cdot 10^5\right]$ MPa,

$A_0 \in [0.0009, 0.0011]\, m^2$, $A_i \in [A_0] \cdot i$ i=1,2,3,4, P=30 kN,

$\varsigma = 7 \cdot 10^{10}$ MPa, $\varsigma_{kin} = 0.1\varsigma$, $\varsigma_{iso} = 0.9\varsigma$, $\sigma_0 = 190$ MPa.

$$\frac{\partial q_i}{\partial h_j} \approx \frac{q_i(\dots, h_j + \Delta h_j, \dots) - q_i(\dots, h_j, \dots)}{\Delta h_j}$$

	q_i [m]
q_1	[0.001148, 0.006707]
q_2	[0.002296, 0.013414]
q_3	[0.002870, 0.016790]
q_4	[0.003252, 0.019056]

PRZEDZIAŁOWY TEST MONOTONICZNOŚCI

$$\hat{\mathbf{K}}([\boldsymbol{\lambda}])\mathbf{q} = \hat{\mathbf{Q}}([\boldsymbol{\lambda}])$$

$$\hat{\mathbf{q}}([\mathbf{h}]) = \text{hull}\,\sum\left(\hat{\mathbf{K}}([\mathbf{h}]), \hat{\mathbf{Q}}([\mathbf{h}])\right)$$

$$\hat{\mathbf{K}}([\boldsymbol{\lambda}])\frac{\partial \mathbf{q}}{\partial \mathbf{h}} = \frac{\partial \hat{\mathbf{Q}}([\boldsymbol{\lambda}])}{\partial \mathbf{h}} - \frac{\partial \hat{\mathbf{K}}([\boldsymbol{\lambda}])}{\partial \mathbf{h}}\hat{\mathbf{q}}([\mathbf{h}])$$

$$\frac{\partial \hat{\mathbf{q}}([\mathbf{h}])}{\partial \mathbf{h}} = \text{hull}\,\sum\left(\hat{\mathbf{K}}([\mathbf{h}]), \frac{\partial \hat{\mathbf{Q}}([\mathbf{h}])}{\partial \mathbf{h}} - \frac{\partial \hat{\mathbf{K}}([\mathbf{h}])}{\partial \mathbf{h}}\hat{\mathbf{q}}([\mathbf{h}])\right)$$

$$[\mathbf{h}] = \bigcup_{\mathrm{i}} [\mathbf{h}_{\mathrm{i}}], \qquad \frac{\partial \hat{\mathbf{q}}([\mathbf{h}])}{\partial \mathbf{h}} \subseteq \bigcup_{\mathrm{i}} \frac{\partial \hat{\mathbf{q}}([\mathbf{h}_{\mathrm{i}}])}{\partial \mathbf{h}}$$

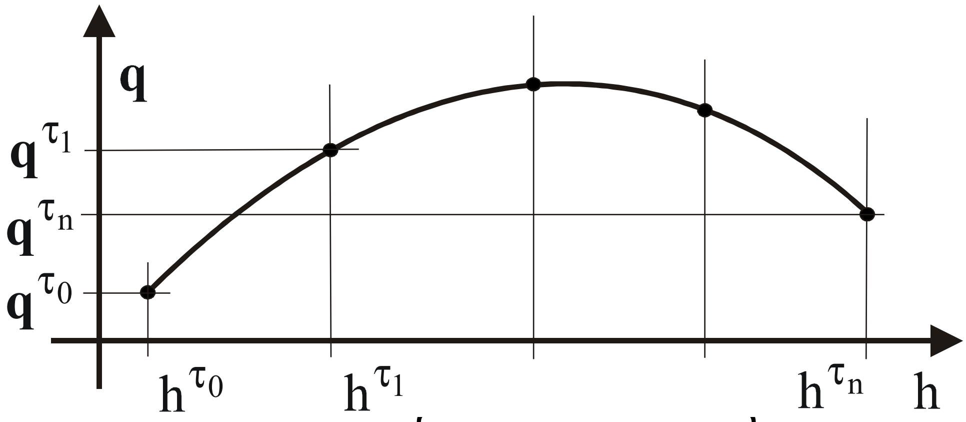
$$\begin{cases} R_1 < r < R_2 : \dfrac{1}{r}\dfrac{d}{dr}\left(r\lambda\dfrac{dT(r)}{dr}\right) + Q = 0 \\ r = R_1 \quad : -\lambda\dfrac{dT(r)}{dr} = \alpha\left(T(r) - T_b\right) \\ r = R_2 \quad : T(r) = T_t \end{cases}$$

R_1=0.0005 [m], R_2=10*R_1, α=2000, T_b= 32 [^{o}C], T_t=37[^{o}C],Q=10245[W/m^3]m, $\lambda \in$ [0.21,0.23][W/mK].

	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
$\underline{T}_i$ [^{o}C]	36.586	35.470	34.782	34.284	33.894	33.573	33.302	33.065	32.857	32.669	32.500
$\overline{T}_i$ [^{o}C]	36.619	35.494	34.800	34.298	33.905	33.582	33.308	33.070	32.859	32.671	32.500

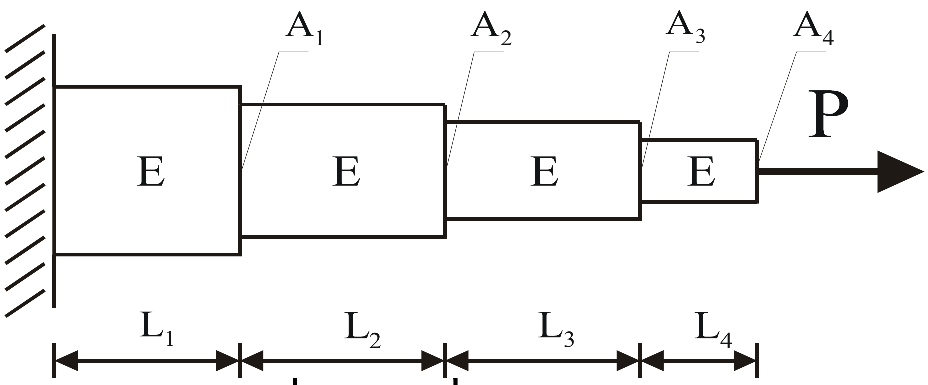
METODA KONTYNUACJI

$$\begin{cases} \mathbf{F}(\mathbf{q}, h) = \mathbf{0} \\ h - h^{\tau} = 0 \end{cases}$$



$$q_i^- \approx \min\left\{q_i^{\tau_0}, q_i^{\tau_1}, \dots, q_i^{\tau_n}\right\}$$

$$q_i^+ \approx \max\left\{q_i^{\tau_0}, q_i^{\tau_1}, \dots, q_i^{\tau_n}\right\}$$



$$P \in \left[P^-, P^+\right] = [25, 30]\,\text{kN}$$

P_1	P_2	P_3	P_4	P_5
0.003750	0.003900	0.004050	0.004200	0.004500

$$P^- = \min_i\{q(P_i)\} = 0.003750\text{ m}$$

$$P^+ = \max_i\{q(P_i)\} = 0.004500\text{ m}$$

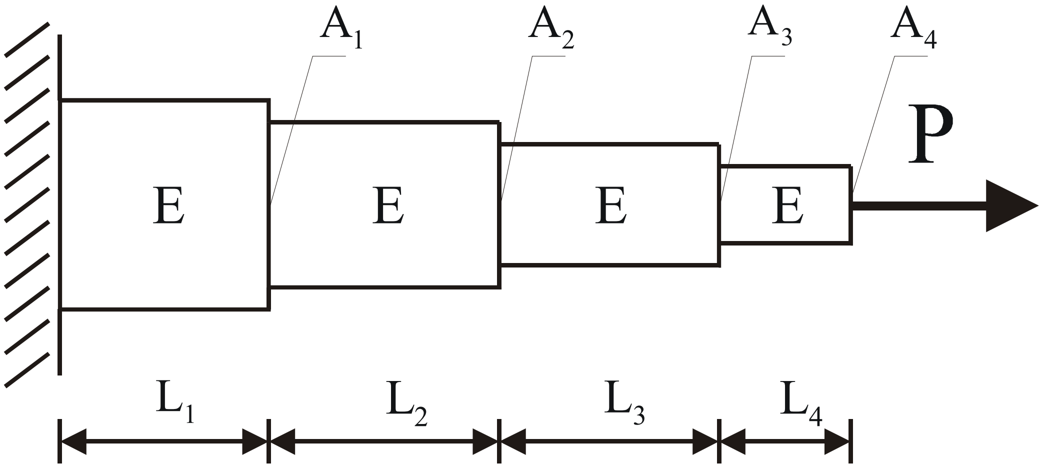
ANALIZA WRAŻLIWOŚCI

$$\frac{\partial q_i}{\partial h_j} \approx \frac{q_i(\ldots, h_j + \Delta h_j, \ldots) - q_i(\ldots, h_j, \ldots)}{\Delta h_j}$$

Metoda wymaga rozwiązania zadania **m+1** razy
(m – jest liczbą przedziałowych parametrów)

$$\frac{\partial q_i^{\tau+\Delta\tau}}{\partial h_j} = \frac{\partial q_i^{\tau}}{\partial h_j} + \left(\frac{\partial \Delta^{\tau} q_i}{\partial h_j} \right)$$

Metoda wymaga rozwiązania **m** zadań



	q_i [m]
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q_4	[0.003252, 0.019056]

WYKORZYSTANIE ISTNIEJĄCEGO OPROGRAMOWANIA MES

$$q_i^- = \min_i \{q_i(h_1), \ldots, q_i(h_k)\}, \quad q_i^+ = \max_i \{q_i(h_1), \ldots, q_i(h_k)\}$$

$$\mathbf{q}(\ldots, h_i + \Delta h_i, \ldots) = \mathbf{K}(\ldots, h_i + \Delta h_i, \ldots)^{-1} \cdot \mathbf{Q}(\ldots, h_i + \Delta h_i, \ldots)$$

$$\frac{\partial q_i}{\partial h_j} \approx \frac{q_i(\ldots, h_j + \Delta h_j, \ldots) - q_i(\ldots, h_j, \ldots)}{\Delta h_j}$$

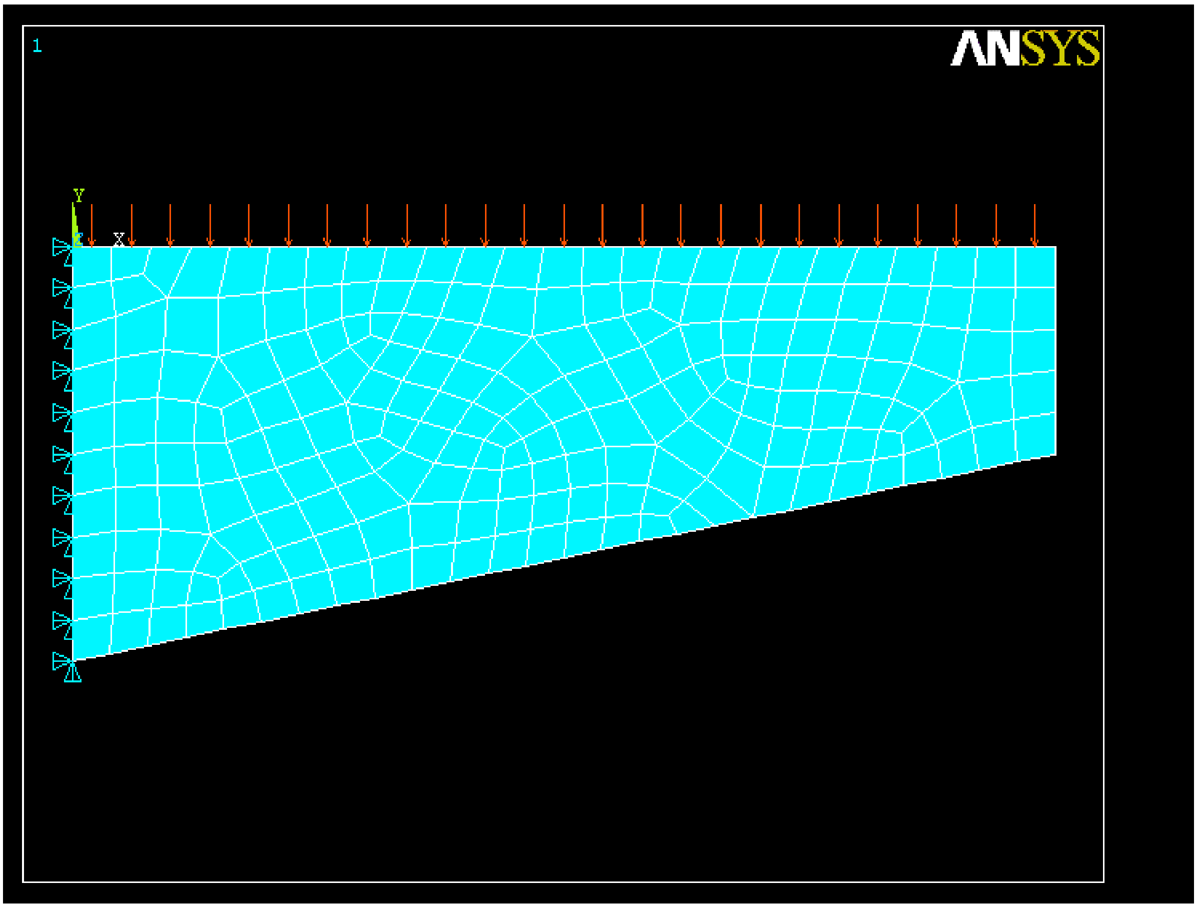
$$\mathbf{K}(\mathbf{h}) \frac{\partial \mathbf{q}}{\partial \mathbf{h}} = \frac{\partial \mathbf{Q}(\mathbf{h})}{\partial \mathbf{h}} - \frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}} \mathbf{q}(\mathbf{h})$$

$$\frac{\partial q_i^{\tau+\Delta\tau}}{\partial h_j} = \frac{\partial q_i^{\tau}}{\partial h_j} + \left(\frac{\partial \Delta^{\tau} q_i}{\partial h_j} \right)$$

$$^{\tau+\Delta\tau}\mathbf{K}(\mathbf{h}) \frac{d^{\tau+\Delta\tau}\Delta\mathbf{q}}{dh} = \left. \frac{d^{\tau+\Delta\tau}\mathbf{R}}{dh} \right|_{\Delta\mathbf{q} \neq \Delta\mathbf{q}(h)}$$

$$\left. \frac{d^{\tau+\Delta\tau}\mathbf{R}}{dh} \right|_{\Delta\mathbf{q} \neq \Delta\mathbf{q}(h)} = \frac{\partial^{\tau+\Delta\tau}\mathbf{Q}}{\partial h} - \int_{\Omega} \mathbf{B}^T \left(\frac{\partial^{\tau}\boldsymbol{\sigma}}{\partial h} + \left. \frac{\partial^{\tau}\Delta\boldsymbol{\sigma}}{\partial h} \right|_{\Delta\boldsymbol{\varepsilon} \neq \Delta\boldsymbol{\varepsilon}(h)} \right) d\Omega$$

WYKORZYSTANIE ISTNIEJĄCEGO OPROGRAMOWANIA MES



$$E \in \left[2.0 \cdot 10^5, 2.2 \cdot 10^5\right] \text{MPa}, \quad \nu \in [0.25, 0.3], \quad q = 10\,\frac{\text{kN}}{\text{m}}$$

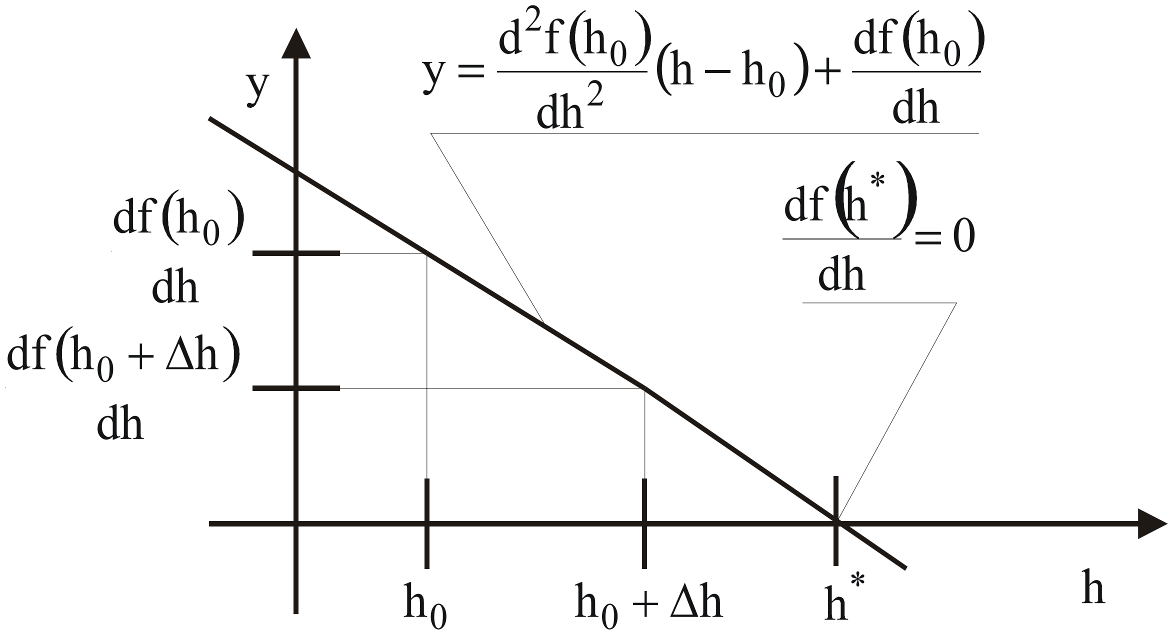
$$\frac{\partial \bar{\sigma}_y(h)}{\partial h} \approx \frac{\bar{\sigma}_y(h+\Delta h) - \bar{\sigma}_y(h)}{\Delta h}$$

$$\frac{\partial u_y^{27}(h)}{\partial h} \approx \frac{u_y^{27}(h+\Delta h) - u_y^{27}(h)}{\Delta h}$$

$$(\bar{\sigma}_y)_{max} \in [0.22814, 0.22836] \text{ MPa}$$

$$u_y^{27} \in [-0.96296\text{E-}05, -0.87810\text{E-}05] \text{ m}$$

DOKŁADNOŚĆ METOD ANALIZY WRAŻLIWOŚCI



$$\left| h_0 - h^* \right| = \left| \frac{\frac{df(h_0)}{dh}}{\frac{d^2 f(h_0)}{dh^2}} \right| >> h^+ - h^-$$

$$\frac{df(h)}{dh} \approx \frac{f(h + \Delta h) - f(h)}{\Delta h}$$

$$\frac{d^2 f(h)}{dh^2} \approx \frac{f(h - \Delta h) - 2f(h) + f(h + \Delta h)}{(\Delta h)^2}$$

$$\left| \frac{[f(h + \Delta h) - f(h)]\Delta h}{f(h - \Delta h) - 2f(h) + f(h + \Delta h)} \right| >> h^+ - h^-$$

TESTY MONOTONICZNOŚCI

$$0 \notin \frac{d\hat{f}([x])}{dx} \quad \Rightarrow \quad f(x) \text{ jest monotoniczna w } [x]$$

$$\text{np. } \mathbf{K}(\mathbf{h})\frac{\partial \mathbf{q}(\mathbf{h})}{\partial \mathbf{h}} = \frac{\partial \mathbf{Q}(\mathbf{h})}{\partial \mathbf{h}} - \frac{\partial \mathbf{K}(\mathbf{h})}{\partial \mathbf{h}}\mathbf{q}(\mathbf{h})$$

$$0 \notin \frac{\partial \hat{q}_i([\mathbf{h}])}{\partial h_j} = \text{hull}\sum\left(\hat{\mathbf{K}}([\mathbf{h}]), \frac{\partial \hat{\mathbf{Q}}([\mathbf{h}])}{\partial \mathbf{h}} - \frac{\partial \hat{\mathbf{K}}([\mathbf{h}])}{\partial \mathbf{h}}\hat{\mathbf{q}}([\mathbf{h}])\right)_{ij}$$

$$\mathbf{F}(\mathbf{q}, \mathbf{h}) = \mathbf{0}$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{q}}\frac{\partial \mathbf{q}}{\partial \mathbf{h}} + \frac{\partial \mathbf{F}}{\partial \mathbf{h}} = \mathbf{0}$$

$$\frac{\partial q_i(\mathbf{h})}{\partial h_j} = \left|\frac{\partial(F_1, \ldots, F_n)}{\partial(q_1, \ldots, h_j, \ldots, q_m)}\right| / \left|\frac{\partial(F_1, \ldots, F_n)}{\partial(q_1, \ldots, q_m)}\right|^{-1}$$

$$\frac{\dfrac{\partial(\hat{F}_1, \ldots, \hat{F}_n)}{\partial(q_1, \ldots, q_m)}([\mathbf{q}], [\mathbf{h}])}{\dfrac{\partial(\hat{F}_1, \ldots, \hat{F}_n)}{\partial(q_1, \ldots, h_j, \ldots, q_m)}([\mathbf{q}], [\mathbf{h}])}$$

UKŁADY LINIOWE

$$\mathbf{K}(\mathbf{h})\mathbf{q} = \mathbf{Q}(\mathbf{h})$$
$$\mathbf{K}(\mathbf{h})\mathbf{q} = \mathbf{Q}$$
$$\mathbf{K}\mathbf{q} = \mathbf{Q}(\mathbf{h})$$

$$\frac{\partial q_i(\mathbf{h})}{\partial h_j} = \left|\frac{\partial(F_1,\ldots,F_n)}{\partial(q_1,\ldots,h_j,\ldots,q_m)}\right| / \left|\frac{\partial(F_1,\ldots,F_n)}{\partial(q_1,\ldots,q_m)}\right|^{-1}$$

$$\mathbf{F} = \mathbf{K}\mathbf{q} - \mathbf{Q}(\mathbf{h})$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{h}} = -\frac{\partial \mathbf{Q}(\mathbf{h})}{\partial \mathbf{h}}$$

$$\left|\frac{\partial \mathbf{F}}{\partial \mathbf{q}}\right| = |\mathbf{K}| = \mathrm{const}$$

Jeśli $Q_i = \sum_j \alpha_{ij} P_j$, to $\dfrac{\partial F_i}{\partial h_j} = -\dfrac{\partial Q_i}{\partial P_j} = -\alpha_{ij} = \mathrm{const}$

$$\left|\frac{\partial(\hat{F}_1,\ldots,\hat{F}_n)}{\partial(q_1,\ldots,h_j,\ldots,q_m)}\right| = \mathrm{const}$$

Przemieszczenia w liniowych układach mechanicznych są monotoniczną funkcją obciążeń

REGULARNOŚĆ MACIERZY PRZEDZIAŁOWYCH

$$\frac{\partial q_i(\mathbf{h})}{\partial h_j} = \left|\frac{\partial\left(\hat{F}_1,\ldots,\hat{F}_n\right)}{\partial\left(q_1,\ldots,h_j,\ldots,q_m\right)}\right| / \left|\frac{\partial\left(\hat{F}_1,\ldots,\hat{F}_n\right)}{\partial\left(q_1,\ldots,q_m\right)}\right|^{-1}$$

$$[\mathbf{A}] \in IR^{n\times n}$$

$$[a_{ij}] = [a_{ij}] - \frac{[a_{is}]}{[a_{ss}]} \cdot [a_{sj}]$$

$$\det(\mathbf{A}) = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$$

$$[\mathbf{q}] = \bigcup_{i=1}^{n_q} [\mathbf{q}_i], \qquad [\mathbf{h}] = \bigcup_{i=1}^{n_h} [\mathbf{h}_i]$$

Jeśli funkcja $q_i(\mathbf{h})$ jest monotoniczna we wszystkich przedziałach $[\mathbf{h}_i]$, to jest monotoniczna w przedziale $[\mathbf{h}]$. (Jeśli znak pochodnej $\frac{\partial q_i(\mathbf{h})}{\partial h_j}$ się nie zmienia.)