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MODELING OF STRUCTURES WITH TAKING INTO ACCOUNT UNCERTAINTY OF THE PARAMETERS

Summary. All parameters of mechanical systems are known with certain greater or smaller accuracy. In some cases the influence of uncertainties is so big, that can not be omitted in process of project design. This can happen e.g. in wall structures, concrete structures, composite structures and in case of wind and snow loads. In many cases we don't have sufficient amount of information to calculation of probabilistic characteristics of constructions. We are enforced in this situation to use the semi-probabilistic methods.
In this paper a new method of calculation of safety of construction will be presented. Uncertainty of some parameters is modeled by using probabilistic methods, and some of the parameters are characterized by using of the safety factor.

MODELOWANIE KONSTRUKCJI Z UWZGLĘDNIENIEM NIEPEWNOŚCI PARAMETRÓW.

Streszczenie. Wszystkie parametry układów mechanicznych są znane z większą lub mniejszą dokładnością. W niektórych przypadkach wpływ niepewności jest na tyle duży, że nie może zostać pominięty w procesie projektowania. Taka sytuacja ma miejsce np. w konstrukcjach murowych, betonowych kompozytowych oraz w przypadku obciążeń wiatrem i śniegiem. W wielu przypadkach nie mamy dostatecznej ilości informacji do obliczenia probabilistycznych charakterystyk konstrukcji. W takich przypadkach jesteśmy zmuszeni wykorzystywać metody półprobabilistyczne.
W pracy przedstawiono nowy algorytm obliczania bezpieczeństwa konstrukcji. Niepewności niektórych parametrów są modelowane przy wykorzystaniu metod probabilistycznych, a inne przy pomocy współczynników bezpieczeństwa.

1 INTRODUCTION

Let us consider the structure which is shown in the Fig. 1. In calculation we assume the following numerical data $L = 1\,[m]$, $P_1^0 = 1012\,[N]$, $P_2 = 500\,[N]$. Now we can calculate the bending moment

$$M(x) = P_2 \cdot (2L - x) - P_1 \cdot (L - x) \cdot H(L - x) \tag{1}$$

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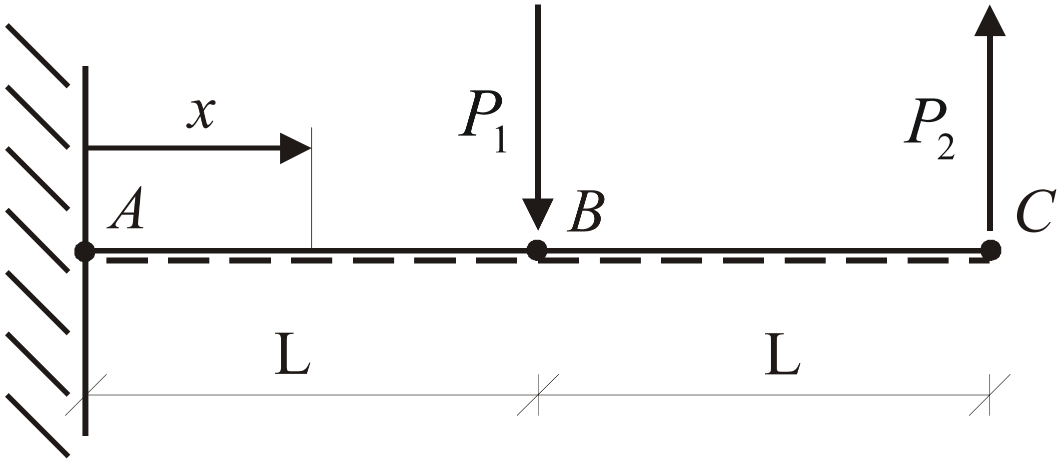

Fig. 1

where

$$H(x)=\begin{cases}1, & x\geq 0\\ 0, & x<0\end{cases} \tag{2}$$

Let's assume that the force P_1 is uncertain and belongs to the following interval:

$$P_1\in\hat{P}_1=\left[P_1^-,P_1^+\right]=\left[P_1^0-0.025\cdot P_1^0,\ P_1^0+0.025\cdot P_1^0\right] \tag{3}$$

$$P_1\in\hat{P}_1=[986.7,1037.3][N] \tag{4}$$

Now we can calculate the interval bending moment

$$\hat{M}(x)=[M^-(x),M^+(x)] \tag{5}$$

where

$$M^-(x)=P_2\cdot(2L-x)-P_1^+\cdot(L-x)\cdot H(L-x) \tag{6}$$

$$M^+(x)=P_2\cdot(2L-x)-P_1^-\cdot(L-x)\cdot H(L-x) \tag{7}$$

and the relative error is the following

$$RelativeError(x)=\frac{M^+(x)-M^-(x)}{M(x)} \tag{8}$$

The relative error is shown in the Fig. 2.

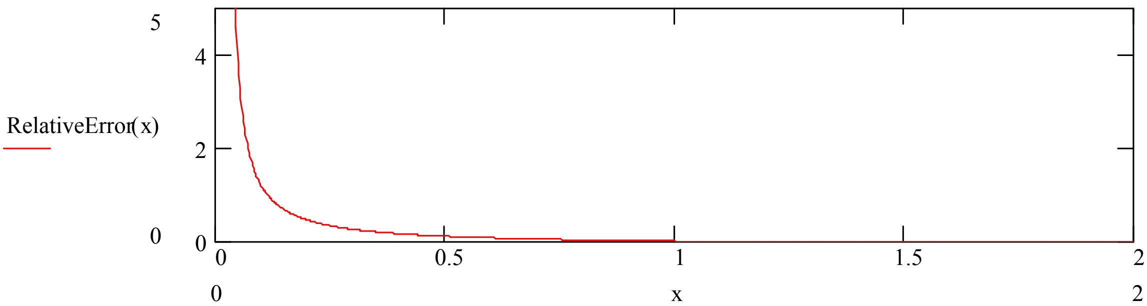

Fig. 2

The maximum relative error is equal to $RelativeError(0) \cdot 100\% = 421.7\%$. The consequence of this error may be very serious.
Let's assume that someone designs a structure which is shown in the Fig. 1 and uses an optimal cross-section. The exact value of cross-section can be calculated from the following equation:

$$|\sigma| \le \sigma_{dop} \Rightarrow \frac{|M(x)|}{W_z} \le \sigma_{dop} \Rightarrow \frac{|M(x)|}{\frac{bh^2}{6}} \le \sigma_{dop} \Rightarrow \sqrt{\frac{6 \cdot |M(x)|}{b}} \le h \tag{9}$$

The relative error of the parameter h (the high of the rectangular cross-section) can be calculated form the equation (9).

$$\frac{\Delta h(x)}{h(x)} = \frac{h^+(x) - h^-(x)}{h(x)} = \frac{\left| \sqrt{\frac{6 \cdot |M^+(x)|}{b}} - \sqrt{\frac{6 \cdot |M^-(x)|}{b}} \right|}{\sqrt{\frac{6 \cdot |M(x)|}{b}}} = \frac{\left| \sqrt{|M^+(x)|} - \sqrt{|M^-(x)|} \right|}{\sqrt{|M(x)|}} \tag{10}$$

$$\frac{\Delta h(x)}{h(x)} = \frac{\Delta \sqrt{|M(x)|}}{\sqrt{|M(x)|}} \tag{11}$$

The maximum relative error in calculation of the area of the rectangular cross-section is equal to

$$\frac{\Delta h(0)}{h(0)} \cdot 100\% = \frac{\Delta \sqrt{|M(0)|}}{\sqrt{|M(0)|}} \cdot 100\% = 71\% . \tag{12}$$

We can see that this error is very big and it cannot be neglected in the calculations.

2 MODELLING OF UNCERTAINTY BY USING THE SET VALUED RANDOM VARIABLE

Sometimes we cannot measure the exact values of physical quantities h but we can estimate the interval $\hat{h} = [h^-, h^+]$ such that:

$$h^- \le h \le h^+ \tag{13}$$

In this case we don't need any information about probability density function. We need only two numbers h^-, h^+ (or some set). We can repeat this procedure for each measurement ω_i finally we get the following interval-valued (or set-valued) random variable

$$\hat{H}_\Omega : \Omega \ni \omega \to \hat{H}_\Omega(\omega) \subset R \tag{14}$$

or more general

$$\hat{\mathbf{H}}_\Omega : \Omega \ni \omega \to \hat{\mathbf{H}}_\Omega(\omega) \subset R^m \tag{15}$$

Traditional (i.e. real-valued) random variable is a special case in this theory.

If this random variable satisfies the following condition

$$\hat{\mathbf{H}}_{\Omega}(\omega_1) \supseteq \hat{\mathbf{H}}_{\Omega}(\omega_2) \supseteq \ldots \supseteq \hat{\mathbf{H}}_{\Omega}(\omega_n) \tag{16}$$

then the set-valued random variable is equivalent to some fuzzy set *F,* which can be defined in the following way [1, 2]:

$$\mu(\mathbf{h} \mid F) = P_{\Omega}\{\omega : \mathbf{h} \in \hat{\mathbf{H}}_{\Omega}(\omega)\} \tag{17}$$

When the data are very uncertain we can assume that

$$\mathbf{H} = conv\{\hat{\mathbf{H}}_{\Omega}(\omega) : \omega \in \Omega\} \tag{18}$$

i.e. the set $\mathbf{H}$ is a convex hull, which contains the all measurements (points, intervals or sets).
All particular cases are shown in the Fig.

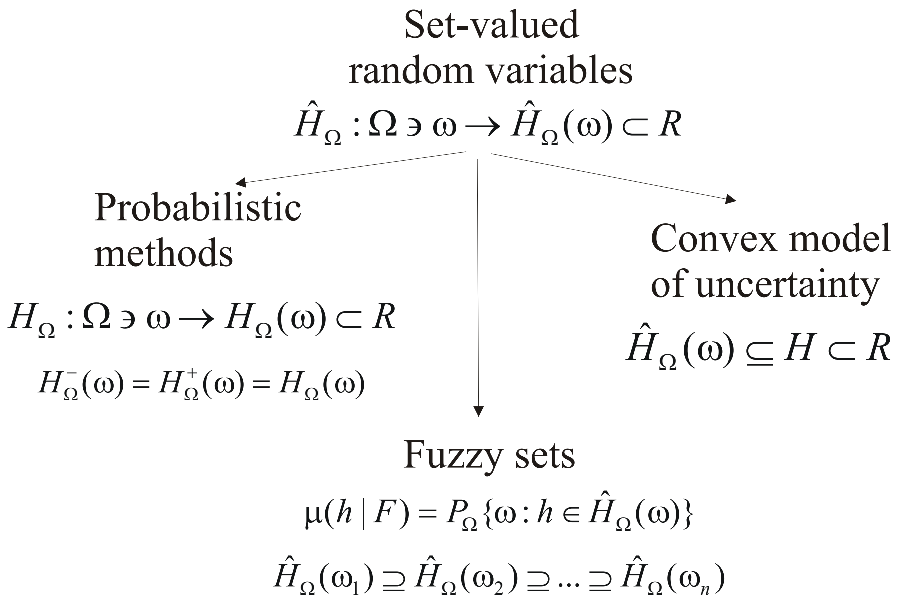

Fig. 3

3 UPPER AND LOWER PROBABILITY

If we have uncertain data only upper and lower probability can be calculated. Upper probability can be defined in the following way

$$Pl\{A\} = P_{\Omega}\{\omega : \hat{\mathbf{H}}_{\Omega}(\omega) \cap A \neq \emptyset\} \tag{19}$$

where $A \in 2^{R^m}$. Lower probability can be defined as follows

$$Bel\{A\} = P_{\Omega}\{\omega : \hat{\mathbf{H}}_{\Omega}(\omega) \subseteq A\} \tag{20}$$

It can be shown that

$$\forall A \in 2^{R^m}, \quad Bel\{A\} \leq P_{\Omega}\{A\} \leq Pl\{A\} \tag{21}$$

Let us consider the structure with pure random $\mathbf{R}_\Gamma : \Gamma \ni \gamma \to \mathbf{R}_\Gamma(\gamma) \in R^n$, interval-valued (set-valued) random $\hat{\mathbf{H}}_\Omega : \Omega \ni \omega \to \hat{\mathbf{H}}_\Omega(\omega) \subset R^m$ and some convex set **H**. The displacements of this structure **u** are the functions of the position of the point $\mathbf{x} \in R^{n_x}$, actual value of the random parameter $\mathbf{R}_\Gamma(\gamma)$ and the actual value of the interval-valued random variable $\hat{\mathbf{H}}_\Omega(\omega)$ and the uncertain set-valued parameter $\mathbf{c} \in \mathbf{C} \subset R^{n_C}$

$$\mathbf{u} = \mathbf{u}(\mathbf{x}, \mathbf{R}_\Gamma(\gamma), \hat{\mathbf{H}}_\Omega(\omega), \mathbf{c}). \tag{22}$$

The solution $\mathbf{u}(\mathbf{x})$ can be described by using upper probability in the following way:

$$Pl\{\mathbf{y} \in \mathbf{u}(\mathbf{x})\} = P_{\Gamma\times\Omega}\{(\omega,\gamma) : \mathbf{y} \in \mathbf{u}(\mathbf{x}, \mathbf{R}_\Gamma(\gamma), \hat{\mathbf{H}}_\Omega(\omega), \mathbf{c}), \mathbf{c} \in \mathbf{C}\}. \tag{23}$$

In can be shown, that from computational point of view, the semiprobabilistic methods are special case of the interval methods [3]. By using the equation (23) we can extend existing civil engineering codes (which are based on semi probabilistic methods) to the calculations which are based on random and set-valued random (and fuzzy) parameters.

4 SOLUTION OF THE EQUATIONS WITH INTERVAL PARAMETERS

In order to check the condition

$$\mathbf{y} \in \mathbf{u}(\mathbf{x}, \mathbf{R}_\Gamma(\gamma), \hat{\mathbf{H}}_\Omega(\omega), \mathbf{c}), \mathbf{c} \in \mathbf{C}. \tag{24}$$

we have to calculate upper and lower bound of the function **u**. If the intervals $\hat{\mathbf{H}}_\Omega(\omega)$ are sufficiently small, then we can find the solution of the equations with interval parameters by using sensitivity analysis. The algorithm is as follows:

Algorithm

1) Formulate parameter dependent system of equation with interval parameters in the form and calculate $\mathbf{u}(\mathbf{h}_\alpha^0)$

$$\mathbf{K}(\mathbf{h}_\alpha^0)\mathbf{u}(\mathbf{h}_\alpha^0) = \mathbf{Q}(\mathbf{h}_\alpha^0), \quad \mathbf{h}_\alpha^0 = mid(\hat{\mathbf{h}}_\alpha), \tag{25}$$

2) For i=1,...,m calculate $\dfrac{\partial \mathbf{u}(\mathbf{h}_\alpha^0)}{\partial h_i}$

$$\mathbf{K}(\mathbf{h}_\alpha^0)\frac{\partial \mathbf{u}(\mathbf{h}_\alpha^0)}{\partial h_i} = \frac{\partial \mathbf{Q}(\mathbf{h}_\alpha^0)}{\partial h_i} - \frac{\partial \mathbf{K}(\mathbf{h}_\alpha^0)}{\partial h_i}\mathbf{u}(\mathbf{h}_\alpha^0). \tag{26}$$

3) For i=1,...,n (n – number of degree of freedom) calculate the sign vector

$$\mathbf{S}_\alpha^i. = \left[sign\left(\frac{\partial u_i(\mathbf{h}_\alpha^0)}{\partial h_1}\right) \quad \ldots \quad sign\left(\frac{\partial u_i(\mathbf{h}_\alpha^0)}{\partial h_m}\right)\right] \tag{27}$$

4) Calculate the independent sign vectors $IndSign_\alpha = \{\mathbf{S}_\alpha^{1*}, ..., \mathbf{S}_\alpha^{k*}\}$ and create vector **U** such, that

$$\mathbf{S}_{\alpha}^{i}=\mathbf{S}_{\alpha}^{j*},\text{ where } j=U_i,\ i=1,...,n \tag{28}$$

5) For i=1,...,p calculate the interval solution $\hat{\mathbf{u}}_{\alpha}^{i*}$

$$\hat{\mathbf{u}}_{\alpha}^{i*}=[\mathbf{u}\left(\mathbf{h}((-1)\mathbf{S}_{\alpha}^{i*},\hat{\mathbf{h}})\right),\mathbf{u}\left(\mathbf{h}(\mathbf{S}_{\alpha}^{i*},\hat{\mathbf{h}})\right)] \tag{29}$$

$$IndSolutin_{\alpha}=\{\hat{\mathbf{u}}_{\alpha}^{1*},...,\hat{\mathbf{u}}_{\alpha}^{p*}\}\,. \tag{30}$$

6) Calculate the extreme interval solution $\hat{\mathbf{u}}_{\alpha}$.
For i = 1,...,n

$$\hat{u}_{\alpha i}=[u_{\alpha i}^{j*-},u_{\alpha i}^{j*-}],\quad\text{where}\quad j=U_i \tag{31}$$

Computational complexity of this algorithm:
- step 1 – 1 solution systems of equations,
- step 5 – $2\cdot p$ solution systems of equations ($1\le p\le n$).

In presented algorithm we have to calculate a system of the equation between $1+2$ and $1+2\cdot n$ times. The examples of applications of this algorithm are shown in the paper [4].

5 CONCLUSIONS

In some engineering problems the influence of the uncertainty on the design process is very big. Existing civil engineering codes are based on the semiprobabilistic methods of modeling of uncertainty. The theory of the set-valued random variables contains the semiprobabilistic methods, probabilistic methods and fuzzy sets method as special cases. By using the equation (23) random, fuzzy and set-valued parameters can be included to the existing methods of the design of strictures without significant changes.

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